

1. (Complete 1 1/2 inch rule)

# SLIDE RULE MANUAL

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A Text Book for  
use with Standard  
Types of Slide Rules

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*by*  
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**DYMOCK'S**  
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## PREFACE

A SLIDE RULE consists of four portions, two outer sections known as the stock which, on some types of rule are mounted on the same base, and on others are held in fixed relation by means of plates at each end of the rule; while the third, or inner section, called the slide, is free to move lengthwise over the full range of the stock. The fourth portion is the cursor, a rectangle of glass on which is engraved a hair line at right angles to the stock and slide.

The simplest form of rule with two scales on the stock and two on the slide is used for multiplication and division, and for obtaining the square or square root of any number. An additional Cube scale gives the cube and cube root of any number, and a Logarithm scale allows the logarithm or anti-logarithm of any number to be found. A Reciprocal scale, usually to be found on the slide, gives the reciprocal of any number and occasionally assists in multiplication and division. Log-log scales are used in involution and evolution and in obtaining the naperian logarithm or anti-logarithm of any number. Trigonometrical scales are provided to obtain the sine and tangent of any angle, and electrical scales give motor and dynamo efficiencies and voltage drops, with or without temperature corrections.

The beginner should not allow himself to be overawed by the seeming multiplicity of lines and figures, because, as will be shown in the following pages, each of the scales can be mastered with comparative ease, the only requirements being an elementary knowledge of mathematics and much practice. The old adage "practice makes perfect," applies to the

manipulation of slide rules in all but its most literal sense. Although they have been in use for many years, it is surprising how few people realize what valuable help slide rules can give in solving a wide variety of problems. The average person has the impression that, to use a slide rule, one must be a mathematician, and whilst it is conceded that there are many abstract problems which can be solved on a slide rule and which would require a mathematician to do them, it is contended that such problems would only present themselves to a man working on higher mathematics. So, in the compilation of this book an effort has been made to produce a text book that will assist beginners and at the same time open new avenues of usefulness of the slide rule to persons already acquainted with its operation.

## CHAPTER 1.

### MULTIPLICATION

Before dealing with any problems of multiplication or division, first observe closely the *A*, *B*, *C* and *D* scales, as these are the scales used.

It will be noted that *A* and *B* are exactly the same as each other, as also are *C* and *D*.

The *A* and *B* scales are divided into two equal sections, or really logarithmic units, numbered 1-10 and 10-100, and subdivided in such a way as to make it simple to express any particular length in decimals, and let it here be stressed that the very first fundamental in quick and accurate slide rule operation is to master perfectly the value expressed by each of the intervening lines between those marked by actual figures. Also it is frequently necessary to visually subdivide the divisions engraved on the rule, so it will be seen just how important the foregoing is.

The *C* and *D* scales are similar to *A* and *B*, except that they consist of but one logarithmic unit. Adjoining each end of these scales are what are known as extensions or super scales; these scales are useful on those occasions when an answer is required at a point just beyond the scale proper, thus avoiding the necessity of moving the slide to a new position.

At certain points along the *A*, *B*, *C* and *D* scales are marks known as constants, namely  $\pi$ -*M*-*C*-*C*<sup>2</sup>; however, further mention of these will be made later on.

In solving problems of multiplication by logarithms, the logarithms of the numbers are added together and an anti-logarithm taken. As the scales on a slide rule are logarithmic, multiplication is performed by adding the lengths of scale representing the value of the numbers concerned. The markings on the scales used are such that the answer can be

read directly, therefore it is not necessary to obtain the anti-logarithm.

EXAMPLE :  $25 \times 3 = 75$

Set 1 on the *C* scale to 2.5, *i.e.*, 25 on the *D* scale, and move the cursor hair line along to 3 on the *C* scale, and read off the answer 7.5, *i.e.*, 75 on the *D* scale. (Fig. 1.)

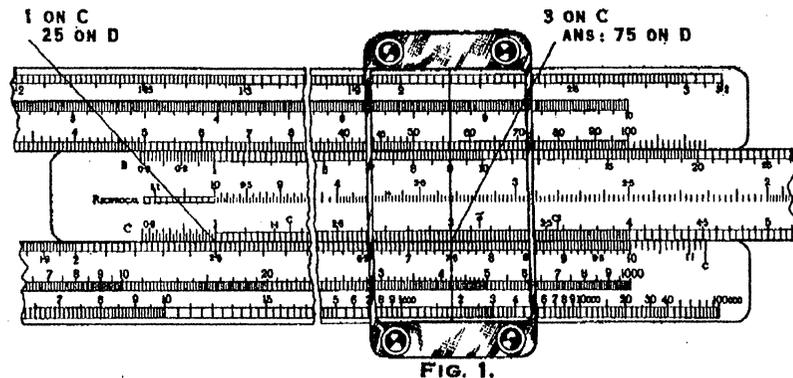


FIG. 1.

A distance corresponding to the logarithm of 25 has been added to a distance corresponding to the logarithm of 3. The combined distance, of course, corresponds to the logarithm of 75.

It will be noted that 2.5 as marked on the rule can represent 2.5, .25, 25, 250 or any other number which, multiplied or divided by 10, or a multiple of 10, gives a result of 2.5.

This principle applies to all numbers on the *A*, *B*, *C*, *D*, or *Reciprocal* scales, so in all further examples on these scales only the actual number in the problem under consideration will be mentioned, it being understood that the position of the decimal point is ignored as far as the location, or reading off, of the number on the scale is concerned.

EXAMPLE :  $1.8 \times 5.5 = 9.9$

Placing 1 of the *C* scale on 1.8 of the *D* scale, and bringing the hair line along to 5.5 on *C*, read off the answer 9.9 on *D*.

It becomes apparent now that had the multiplier been, say, 7, the answer could not have been found on the *D* scale with the rule in its present setting. There are two methods of overcoming this seeming difficulty. Firstly, the problem could have been worked out on the *A* and *B* scales, following the same method as above, *i.e.*, place 1 of the *B* scale on 1.8 of the *A* scale, and read off opposite 7 on the *B* scale the answer 12.6 on the *A* scale.

The other alternative would be to place 10 on the *C* scale over 1.8 on the *D* scale, realizing that now 1 on the *C* scale is also over 1.8 on an imaginary continuation to the left of the *D* scale, so that coming along to the multiplier, 7, on the *C* scale, opposite this on the *D* scale is the answer of 12.6.

Summed up, it will be seen that problems of multiplication are equally capable of solution on either the *A* and *B* or *C* and *D* scales, but, whereas in using *A* and *B* a resetting of the slide is avoided, this fact is compensated for in using the *C* and *D* scales, by the greater measure of accuracy possible by the broader spacing of the logarithmic unit mentioned above.

At this juncture it will be necessary to clarify the subject of where to place the decimal point.

Before laying down any rules for the location of the decimal point, it must be stressed very strongly that, wherever it is possible, locate the decimal point in either multiplication or any other type of problem, by mentally converting the problem into round figures and applying simple arithmetic. This should be done as it is often far more simple than applying a set of rules. But for those occasions when it is not possible to locate the decimal point by sight, it is proposed to lay down a set of rules for each type of problem, commencing with multiplication.

Obviously, the number of whole numbers in the problem has a direct bearing on the number of whole numbers in the result, so a number such as 47 is referred to as being +2, 1.8 as +1, 1054 as +4, .135 as  $\pm 0$ , .026 as -1, .0026 as -2, and so on.

However, there is one other factor to consider, and that is the movement of the slide, or, to be more specific, whether 1 or 10 on the *C* scale is used for the setting. When 1 is used,

subtract one from the sum of the whole numbers, but when the 10 is used, nothing is added nor subtracted.

$$\begin{array}{l} \text{Thus: } 25 \times 3 \\ +2+(+1)=+3 \\ \text{Subtract } \underline{+1} \text{ (because 1 on C was used).} \\ =+2 \text{ or } 75 \cdot 0 \end{array}$$

Before proceeding further, work out the following exercises, using the scales specified. The answers to these and all subsequent exercises, will be found on page 59.

#### EXERCISES ON MULTIPLICATION.

- Exercise 1.  $1 \cdot 4 \times 7$  on the *C* and *D* scales.
2.  $2 \cdot 1 \times 6$  on the *C* and *D* scales.
3.  $340 \times 25$  on the *A* and *B* scales.
4.  $65 \times 40$  on the *A* and *B* scales.
5.  $4 \cdot 05 \times 3 \cdot 9$  on the *C* and *D* scales.

In exercises 2 and 5 it will be noted that it is necessary to move the slide out to the left.

#### CONTINUOUS MULTIPLICATION

The next type of problem to discuss is that of continuous multiplication, *i.e.*, where three or more factors are to be multiplied.

EXAMPLE:  $1 \cdot 8 \times 25 \times 2 \cdot 2 = 99 \cdot 0$

This can be solved by either one of two methods: firstly, using only the *A* and *B* or *C* and *D* scales; secondly, by utilizing what is called the *Reciprocal* scale. First method would be to set 1 on the *C* scale to 1·8 on the *D* scale and bring the cursor line along to 25 on the *C* scale, thus multiplying the first two factors; all that remains to be done is to draw the slide along until 1 on the *C* scale is under the hair

line and then opposite the last multiplier 2·2 on the *C* scale will be found the answer 99·00 on the *D* scale.

As regards the decimal point:

$$\begin{array}{l} 1 \cdot 8 \times 25 \times 2 \cdot 2 \\ +1+(+2)+( +1) = +4 \end{array}$$

Then from this subtract +2 because 1 on the *C* scale was used twice in the working of the problem, thus the answer has a value of +2 or 99·00.

Exactly the same principle applies if the *A* and *B* scales are used.

As a further example solved by this method,

$$14 \times 6 \cdot 5 \times 23 \cdot 6 = 2148$$

Set 1 on the *C* scale to 14 on the *D* scale, and bring the hair line along to 6·5 on the *C* scale. Now it becomes obvious that it would be useless to set 1 on the *C* scale under the hair line, so set 10 under it and then read back to 23·6 on the *C* scale and find the corresponding point on the *D* scale to be 2148.\*

The same principle applies regardless of how many factors there are to be multiplied together. Multiply the first two, then take each succeeding one in turn, until the sum is completed.

The *Reciprocal* scale is used in the second method of continuous multiplication, but only in conjunction with the *C* and *D* scales. This scale, marked *CI* or *CR* or *Reciprocal*, is usually engraved on the slide midway between the *B* and *C* scales, and it will be noted that it is exactly the same as the *C* scale, with the exception that the logarithmic unit is graduated from right to left; thus it becomes the reciprocal or inverse of the *C* scale.

Again solve the example of  $1 \cdot 8 \times 25 \times 2 \cdot 2$ , this time incorporating the *Reciprocal* scale, and take notice that, by so doing, the number of slide settings to perform this multiplication of three factors is reduced to one.

\*The answer to this problem is actually 2147·6, but it must be realized that this degree of accuracy is not possible on a slide rule of convenient length.

Set the hair line at 1.8 on the *D* scale and bring the slide into such a position that 25 on the *Reciprocal* scale corresponds with it, then locate the remaining multiplier 2.2 on the *C* scale, and opposite this point on the *D* scale will be found the answer 99.00 (Fig. 2).

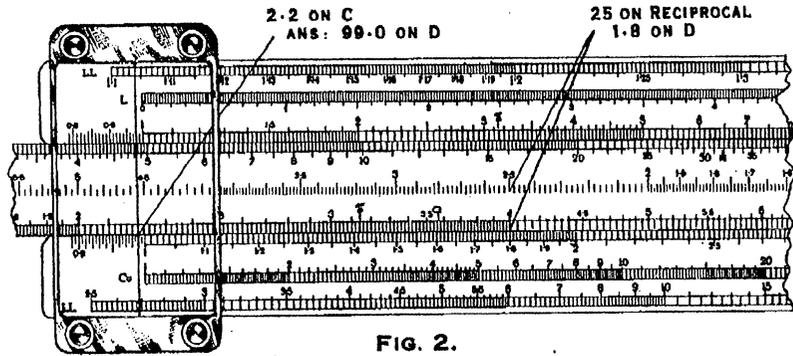


FIG. 2.

It will be noted here how useful the scale extensions can be, because otherwise in the solving of this problem a further setting of the slide would have been necessary.

EXAMPLE :  $43 \times 35 \times 3.92 = 5900$

Line up 43 on the *D* scale with 35 on the *Reciprocal* scale by means of the hair line, and corresponding to the second multiplier, 3.92 on the *C* scale, will be found the answer 5900 on the *D* scale.

EXERCISES ON CONTINUOUS MULTIPLICATION.

Solve these, firstly using only the *C* and *D* scales and then with the *Reciprocal* scale. Valuable practice will be obtained.

- Exercise 6.  $2.5 \times 6 \times 65$
- 7.  $75 \times 1.8 \times 4.5$
- 8.  $.0405 \times 165 \times 8.9$
- 9.  $3.56 \times 2.3 \times 9.05$

CHAPTER 2.

DIVISION

In solving problems of division by logarithms, the logarithmic values are subtracted, so with this fact in mind, solve the following very simple division on the slide rule :

EXAMPLE :  $8 \div 2 = 4$

Set the hair line to 8 on the *D* scale, and bring 2 on the *C* scale along to correspond with it, then transfer the hair line to 1 on the *C* scale and read off opposite this point, the answer 4 on the *D* scale. Closer examination of the rule in this position will reveal that the logarithmic length of the divisor 2 has been subtracted from the logarithmic length of the dividend 8.

EXAMPLE :  $87.5 \div 2.5 = 35$

Set the hair line to 87.5 on the *D* scale, and bring 2.5 on the *C* scale to correspond with it, and on the *D* scale opposite 1 on the *C* scale is the answer 35. (Fig. 3.)

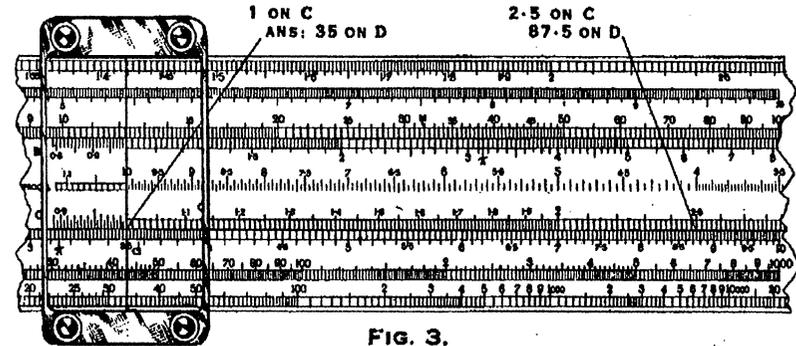


FIG. 3.

EXAMPLE :  $41 \div 5 = 8.2$

Having made the appropriate settings, 5 on the *C* scale to 41 on the *D* scale, it is noted that 1 on the *C* scale is beyond the graduations of the *D* scale, but whereas in multiplication a resetting of the slide is necessary, in division simply read off the answer on the *D* scale under 10 on the *C* scale, *i.e.*, 8.2. Obviously, this 10 bears exactly the same relation to the *D* scale as the 1 on the *C* scale does to an imaginary extension to the left of the *D* scale.

EXAMPLE :  $.066 \div .004 = 16.5$

Set 4 on the *C* scale over 66 on the *D* scale and read off the answer 16.5 on the *D* scale opposite 1 on the *C* scale.

As in multiplication, the *A* and *B* scales can be used instead of the *C* and *D* scales, where it is so desired.

Before proceeding with any exercises, reference may be made to the problem of the location of the decimal point as it applies to division.

When performing multiplication, a number is referred to as being of plus or minus value, depending upon the number of whole numbers it contains, or failing that, the number of noughts immediately following the decimal point. The same principle applies in division, but whereas in multiplication these values were added, it naturally follows that when doing division they will be subtracted, then all that remains is to apply the simple rule that when the answer is read by means of 1 on the *C* scale, add one to the difference, but when it is necessary to use the 10 on the *C* scale, neither add nor subtract anything from the result obtained.

Thus :  $87.5 \div 2.5$

$$+2 - (+1) = +1$$

Add +1 (because the answer was read by using the 1 on the *C* scale).

$$= +2 \text{ or } 35.00$$

Or :  $.066 \div .004$

$$-1 - (-2) = +1$$

Again add 1 because the answer was read opposite the 1 on the *C* scale, thus it is to have two whole numbers on the left side of the decimal point, *i.e.*, 16.5

## EXERCISES ON DIVISION.

Exercise 10.  $78 \div 40$

11.  $9.0 \div .059$

12.  $.280 \div .00415$

13.  $13400 \div 5075$

## CONTINUOUS DIVISION

Continuous division is performed in the same manner as simple division, excepting, of course, that an extra setting is necessary for each successive divisor.

EXAMPLE :  $85 \div 3.24 \div 1.6 = 16.4$

Set the hair line at 85 on the *D* scale and bring 3.24 on the *C* scale to correspond with it, then bring the hair line to 1 on the *C* scale ; the next step being to bring 1.6 on the *C* scale under the hair line and then opposite 1 on the *C* scale, will be found the answer 16.4 on the *D* scale.

As regards the decimal point, the sum is

$$+2 - (+1) - (+1) = \pm 0$$

with +2 to add because 1 on the *C* scale was used twice ; thus the answer has two figures to the left of the decimal.

EXAMPLE :  $46.5 \div 7.1 \div .087 = 75.3$

As before, line up the divisor 7.1 on the *C* scale with the dividend 46.5 on the *D* scale. As the 1 on the *C* scale is beyond the range of the stock, slide the cursor along to the 10 on the *C* scale and perform the second division by bringing .087 on the *C* scale along to the hair line and read off the answer 75.3 on the *D* scale opposite the 10 on the *C* scale.

The position of the decimal is determined by the sum

$$+2 - (+1) - (-1) = +2$$

with nothing to add this time because on both occasions 10 on the *C* scale was used as the focal point.

## EXERCISES ON CONTINUOUS DIVISION.

Exercise 14.  $6.65 \div 3.92 \div 1.79$

15.  $204 \div 37 \div 2.7$

16.  $47 \div 21 \div 3.12$

17.  $552 \div 39 \div 19$

## CHAPTER 3.

### COMBINATION OF MULTIPLICATION AND DIVISION

Having covered fully the methods of performing multiplication and division separately, the next type of problem to consider is one containing a combination of these two factors. This can be achieved by taking each portion of the problem separately and working it out.

THUS IF THE PROBLEM IS  $\frac{35 \times 64}{2 \cdot 6}$

Divide 35 by 2·6 then multiply the result by 64. No notice need be taken of the intermediate result.

EXAMPLE :  $\frac{65 \cdot 6 \times \cdot 048}{3 \cdot 68} = \cdot 856$

Set the hair line to 65·6 on the *D* scale and bring 3·68 on the *C* scale to correspond with it, then transfer the hair line to ·048 on the *C* scale ; and opposite this point on the *D* scale is the answer ·856

EXAMPLE :  $\frac{45 \cdot 5 \times 36 \times \cdot 95}{38 \times 6 \cdot 8} = 6 \cdot 02$

Set the hair line to 45·5 on the *D* scale and bring 38 on the *C* scale to correspond with it, then transfer the hair line to 36 on the *C* scale and the first portion of the problem, *i.e.*,

$$\frac{45 \cdot 5 \times 36}{38}$$

is thus completed. Then divide this by 6·8 by bringing 6·8 of the *C* scale under the hair line. The last multiplication is automatically performed and the answer (6·02) is found on the *D* scale corresponding with the last multiplier ·95 on the *C* scale.

The sum for the decimal point is

$$\begin{array}{r} +2 + (+2) + (\pm 0) = +4 \\ \text{Subtract } +2 + (+1) = +3 \\ \hline = +1 \end{array}$$

The slide movements cancelled each other out, thus the answer has one figure to the left of the decimal, *i.e.*, 6·02.

It will be noted that in solving problems involving a combination of multiplication and division, it is preferable to alternately multiply and divide, rather than complete either one of them before commencing the other.

It is also possible to utilize the *Reciprocal* scale in calculations of this nature ; for instance, in the first example

$$\frac{65 \cdot 6 \times \cdot 048}{3 \cdot 68}$$

Bring 65·6 on the *Reciprocal* scale to correspond with ·048 on the *D* scale using the hair line, then beneath 3·68 on the *Reciprocal* scale will be found the answer ·856 on the *D* scale. (Fig. 4.)

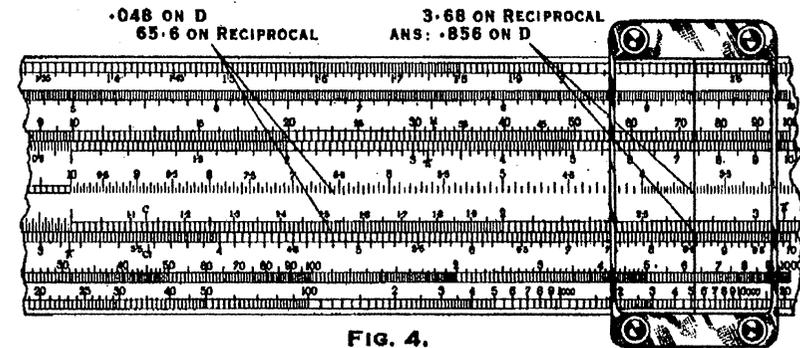


FIG. 4.

The *A* and *B* scales can, of course, be used for these problems combining multiplication and division with the advantage that the need of resetting the slide does not arise, but with the disadvantage of a lesser degree of accuracy than can be obtained from the *C* and *D* scales.

#### EXERCISES ON COMBINATION OF MULTIPLICATION AND DIVISION

Exercise 18.  $\frac{3 \cdot 5 \times 27 \cdot 8}{16 \cdot 8}$

20.  $\frac{15 \times 14 \cdot 8 \times 12}{6 \cdot 48}$

19.  $\frac{42 \times \cdot 75 \times \cdot 02}{1 \cdot 15}$

21.  $\frac{324 \times 15 \cdot 6 \times 2 \cdot 12}{17 \cdot 8}$

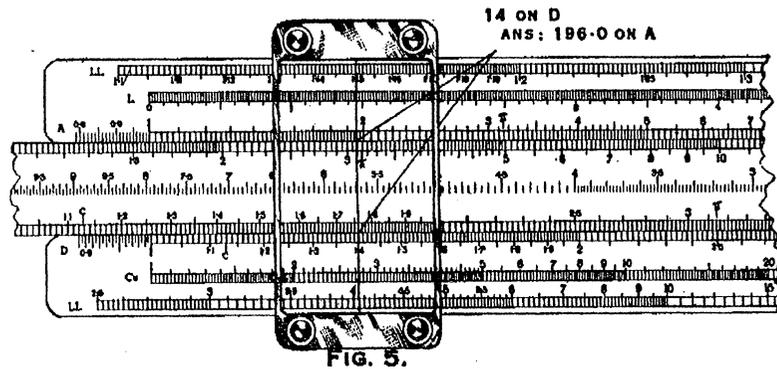
## CHAPTER 4.

### SQUARES

The square or square root of a number can be obtained by a simple method due to the fact that the *A* and *B* scales consist of two logarithmic units, whereas *C* and *D* have only one.

By logarithms  $n^2 = \log n + \log n$  or  $2 \log n$ . Thus the square of any number on the *C* or *D* scales is found directly above it on the *B* or *A* scales respectively, and, of course, to obtain this direct reading the hair line is used.

Thus to find the value of  $14^2$  place the hair line over 14 on the *D* scale and the answer 196.0 will be found on the *A* scale, corresponding with the hair line. (Fig. 5.)



To find the value of  $5.7^2$ , place the hair line over 5.7 on the *D* scale and read off the answer 32.5 on the *A* scale.

Or to find  $.029^2$  set the hair line over .029 and read off the answer .000841.

Somewhat similar to the method of establishing the decimal in multiplication is used for squaring. The rule is to

multiply the number of figures to the left of the decimal point or noughts to the right of the decimal point by 2, and then, if the answer falls in the first section of the *A* or *B* scales, subtract +1, but neither add nor subtract anything if the answer falls in the second section of the *A* or *B* scales.

$$\begin{aligned} .029^2 &= -1 \times 2 = -2 \\ \text{Subtract } +1 & \text{ (because the answer is in the first} \\ & \text{section of A).} \\ &= -3 \text{ or } .000841 \end{aligned}$$

The solution of such problems as  $(2.56 \times 3.1)^2$  is accomplished at a single setting by the following method.

Set 1 on the *C* scale at 2.56 on the *D* scale and, with the hair line at 3.1 on the *C* scale, read off the answer 63.0 on the *A* scale.

The location of the decimal point is determined by the rules relating to multiplication and squaring which apply to the problem in question.

#### EXERCISES ON SQUARES.

- |              |          |     |          |
|--------------|----------|-----|----------|
| Exercise 22. | $5.05^2$ | 24. | $.026^2$ |
| 23.          | $47^2$   | 25. | $1.85^2$ |

## CHAPTER 5.

### SQUARE ROOTS

To obtain the square root of a number the procedure is, of course, just the opposite to that of obtaining the square, the reading being taken from the *A* scale down to the *D* scale, or the *B* scale down to the *C* scale with the aid of the hair line. One of the most important factors in taking a square root of a number is to place it in the correct section of the *A* or *B* scales.

Whereas the square of, say, 6.5 is the same as the square of 65, excepting for the location of the decimal point, the square roots of 6.5 and 65 are vastly different.

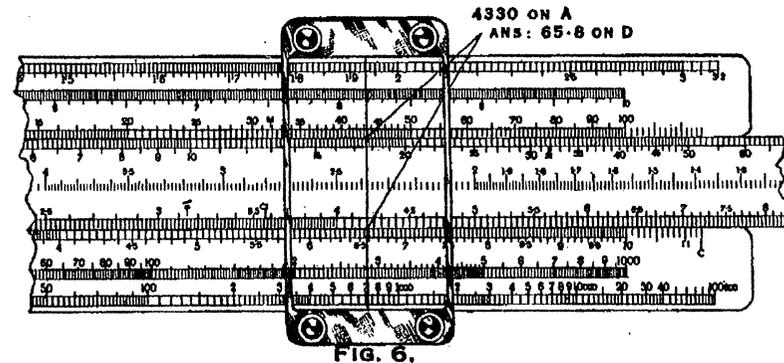
In extracting the square root it would be correct to place 6.5 in the first section of *A*, 65 in the second section, 650 in the first section, 6500 in the second section, and so on.

The following set of rules are to be applied:—

1. If the number be more than unity, mark off the digits in pairs to the left of the decimal, and if the number be less than unity, mark it off to the right of the decimal. Thus 1'01'05.6 or .56'47'2
2. Should the left-hand side group consist of only one significant figure as 1'01'05.6, use the first section of *A*.
3. Should the left-hand side group consist of two significant figures as .56'47'2, use the second section of *A*.
4. The square root of a number above unity will have as many whole numbers as there are groups of two, or parts of groups, contained in the original number, and the square root of a number below unity will have as many noughts immediately following the decimal point as there are groups of noughts immediately following the decimal point in the original number.

EXAMPLE :  $\sqrt{4330.0} = 65.8$

Mark it off thus, 43'30.0, and as the left-hand group consists of two figures—43—therefore, by rule 3, set it on the second section of the *A* scale as 43.30 and on the *D* scale read off 658; as there are two groups to the left of the decimal, the square root will have two numbers to the left of the decimal (rule 4) or 65.8. (Fig. 6.)



EXAMPLE :  $\sqrt{68600.0} = 261.9$

Mark off as 6'86'00.0. The left-hand group in this case consists of but one figure, so place it in the first section of *A* (rule 2) and read off on the *D* scale 2619; and as there are three groups, 6-86-00, the answer has three numbers to the left of the decimal, thus, 261.9

EXAMPLE :  $\sqrt{.000237} = .0154$

Mark off as .00'02'37. By rule 2 use the first section of the *A* scale because the first group to contain a significant figure—02—has, of course, only one such, 2. So, setting the hair line to 237 read off on the *D* scale 154, and as there is only one pair or group of noughts immediately following the decimal point the answer will be .0154

#### EXERCISES ON SQUARE ROOTS.

- |              |                |     |                 |
|--------------|----------------|-----|-----------------|
| Exercise 26. | $\sqrt{550}$   | 28. | $\sqrt{1.69}$   |
| 27.          | $\sqrt{28500}$ | 29. | $\sqrt{.00305}$ |

CHAPTER 6.

CUBES

The cube of a number can be found by using the *A, B, C, D* scales, but there is a more simple method by utilizing a special scale, graduated into three logarithmic units, and placed sometimes on the edge of the rule but usually on the face, and used in conjunction with the *D* scale. Without this special scale the cube of a number can be found in two steps.

EXAMPLE:  $1.8^3 = 5.83$

Place 1 of the *C* scale on 1.8 of the *D* scale and the hair line at 1.8 on the *B* scale; and opposite this on the *A* scale will be found the answer 5.83. Obviously, what has been done is to square 1.8 and multiply the answer by 1.8. (Fig. 7.)

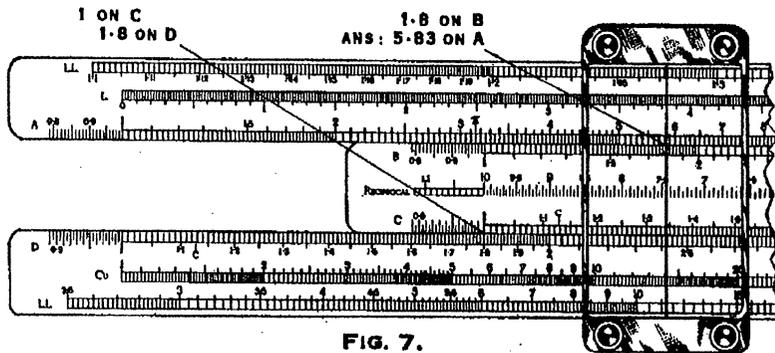


FIG. 7.

However, by using the special scale (*Cu*), cubes or cube roots can be solved with a single setting of the hair line.

Thus to find  $14^3$  set the hair line to 1.4 on the *D* scale and read off 2.744 on the *Cu* scale. (Fig. 8.) As regards the

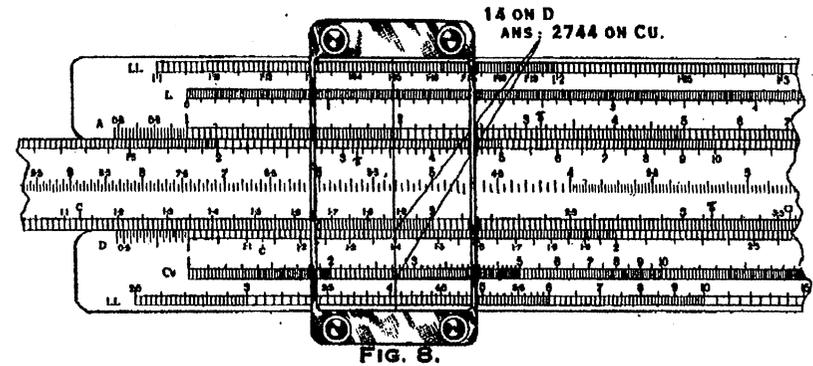


FIG. 8.

decimal point, the rule is to move it in the answer three times the number of places, and in the opposite direction to what it was moved to accommodate it on the *D* scale. Thus the answer to the above example becomes 2744.0.

EXAMPLE:  $410^3 = 69,000,000$

Set the hair line at 4.10 on the *D* scale and read off 69.0 on the *Cu* scale, and as the decimal had to be moved two places to the left, in the answer it must be moved six places to the right, 69,000,000.

EXAMPLE:  $.0132^3 = .0000023$

Set the hair line to 1.32 on the *D* scale and on the *Cu* scale read off 2.3, then, as regards the location of the decimal point, as it was necessary to move it two places to the right it must now be moved six places to the left, thus .0000023

EXERCISES ON CUBES.

- Exercise 30.  $1.6^3$     31.  $27^3$     32.  $.014^3$     33.  $9.8^3$

## CHAPTER 7.

### CUBE ROOTS

In extracting the cube root of a number it must be placed in the correct one of the three logarithmic units which form the *Cu* scale.

1. If the number be more than unity mark off the digits in threes to the left of the decimal, and if the number be less than unity mark it off in threes to the right of the decimal.

2. Should the left-hand side group consist of only one significant figure as 1'456'8 or .005'689, set it on the first section of the *Cu* scale.

3. Should the left-hand side group consist of two significant figures as 14'568'0 or .056'890, use the middle section of the *Cu* scale.

4. Should the left-hand side group consist of three significant figures as 145'680'0 or .568'90, use the third, or right-hand, section of the *Cu* scale.

5. The cube root of a number above unity will have as many whole numbers as there are groups of three, or parts of groups, contained in the original number to the left of the decimal point, and the cube root of a number below unity will have as many noughts immediately following the decimal point as there are groups of noughts immediately following the decimal point in the original number.

EXAMPLE :  $\sqrt[3]{58411.0} = 38.8$

Mark it off thus, 58'411.0, and as the left-hand group consists of but two figures—58—set the hair line over 58.411 on the middle section of the *Cu* scale (rule 3), and on the *D* scale read off 388.

As there are two groups or parts of groups to the left of the decimal, the answer is 38.8 (rule 5).

EXAMPLE :  $\sqrt[3]{.000055} = .038$

Mark it off .000'055. By rule 3 use the middle section of

the *Cu* scale and obtain a reading on the *D* scale of 38, which, by rule 5, becomes .038, as there was one complete group of noughts immediately following the decimal.

Without a special cube scale, cube roots can be obtained by a method of trial and error, using the *A*, *B*, *C*, *D* scales. This method employs the same principle as obtaining the cube on the *A*, *B*, *C*, *D* scales, but the number must be placed in the correct section of the *A* scale, the correct section of the *B* scale used in conjunction with it, and the answer read off, in accordance with the following rules.

6. Should the left-hand side group consist of only one significant figure set it in the left-hand section of the *A* scale, in conjunction with the left-hand section of the *B* scale, and read off the answer on the *D* scale under 1 on the *C* scale.

7. Should the left-hand side group consist of two significant figures set it in the right-hand section of the *A* scale, in conjunction with the left-hand section of the *B* scale, and read off the answer as rule 6.

8. Should the left-hand side group consist of three significant figures, set it in the right hand section of the *A* scale, in conjunction with the right-hand section of the *B* scale, and read off the answer on the *D* scale under the 10 on the *C* scale.

9. Set the hair line to the number on the *A* scale of which the cube root is required, then move the slide until the value on the *B* scale under the hair line is the same as the product of the value on the *D* scale and the index used on the *C* scale.

EXAMPLE :  $\sqrt[3]{58411.0} = 38.8$

Set the hair line to 58.411 on the *A* scale, move the slide along in accordance with rule 7 until the values on the *B* and *D* scales correspond as in rule 9, and obtain the answer of 38.8.

#### EXERCISES ON CUBE ROOTS.

Exercise 34.  $\sqrt[3]{210}$       36.  $\sqrt[3]{42.9}$   
35.  $\sqrt[3]{6.42}$       37.  $\sqrt[3]{.00274}$

CHAPTER 8.

RECIPROCAL

The *Reciprocal* scale was mentioned in performing continuous multiplication and combined multiplication and division. Other uses are:—

The value  $\frac{1}{n}$  can be obtained by a single setting of the hair line over  $n$  on the *C* scale, the corresponding value on the *Reciprocal* scale being the answer required.

EXAMPLE: The reciprocal of 105 = .00952

Set the hair line over 105 on the *C* scale and the corresponding value on the *Reciprocal* scale of 952, actually .00952, is the answer required. (Fig. 9.)

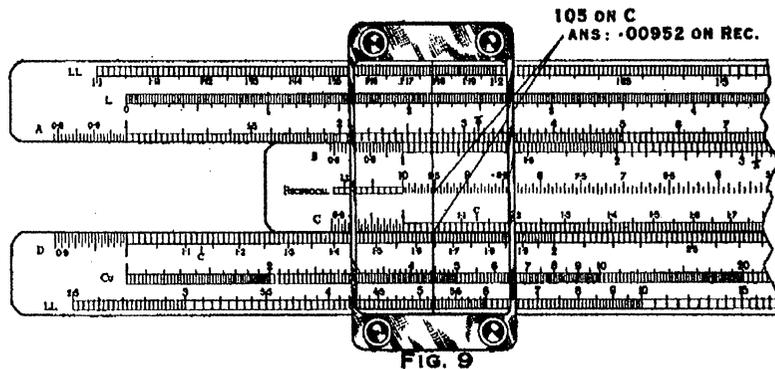


FIG. 9

The rule for the location of the decimal point is that the reciprocal of a number above unity will have one less nought immediately following the decimal than there are figures to the left of the decimal in the original number, thus:

$$\frac{1}{5} = .2 \quad \frac{1}{50} = .02 \quad \frac{1}{500} = .002$$

The reciprocal of a number below unity will have one more figure to the left of the decimal than there are noughts immediately following the decimal in the original number, thus:

$$\frac{1}{.5} = 2.0 \quad \frac{1}{.05} = 20.0 \quad \frac{1}{.005} = 200.0$$

EXERCISES ON  $\frac{1}{n}$

- Exercise 38.  $\frac{1}{275}$       39.  $\frac{1}{.072}$       40.  $\frac{1}{9.64}$

The value  $\frac{1}{n^2}$  is obtained by setting the hair line over  $n$  on the *Reciprocal* scale, and the corresponding value on the *B* scale is the answer required.

EXAMPLES:  $\frac{1}{2^2} = .250$        $\frac{1}{3^2} = .111$   
 $\frac{1}{35 \cdot 2^2} = .0008$        $\frac{1}{.002^2} = 250,000$

The rules regarding the location of the decimal point are those that would apply if the problem was done in two sections, firstly squaring the denominator, then dividing 1 by the result. Taking  $\frac{1}{35 \cdot 2^2}$  as an example, the sum for the location of the decimal would be

$$35 \cdot 2^2 \text{ is } +2 \times 2 = +4$$

Nothing to subtract because the answer falls in the second section of the *A* scale;

$$\text{therefore } 1 \div 35 \cdot 2^2 \text{ is } +1 - (+4) = -3$$

Thus the answer is .0008

Again, taking  $\frac{1}{.002^2}$  the sum for the location of the decimal would be

$$\begin{array}{r} .002^2 \text{ is } -2 \times 2 = -4 \\ \text{Subtract } +1 \text{ (because the answer falls in} \\ \text{the first section of the } A \text{ scale)} \\ \hline = -5 \end{array}$$

therefore  $1 \div .002^2$  is  $+1 - (-5) = +6$

Thus the answer is 250,000.0

#### EXERCISES ON $\frac{1}{n^2}$

Exercise 41.  $\frac{1}{45^2}$       42.  $\frac{1}{.036^2}$       43.  $\frac{1}{2.35^2}$

The value  $\frac{1}{\sqrt{n}}$  is obtained by setting the hair line over  $n$  on the *B* scale (in accordance with rules 2 and 3, page 18), and reading off the answer on the *Reciprocal* scale.

EXAMPLE:  $\frac{1}{\sqrt{94}} = .1032$

The sum for the location of the decimal point is a combination of the rules for taking square roots and for division. Thus, in the above example the value of  $\sqrt{94}$  is +1 (rule 4, Square Roots);

therefore  $1 \div \sqrt{94}$  is  $+1 - (+1) = \pm 0$  or .1032

EXAMPLE:  $\frac{1}{\sqrt{.0056}} = 13.37$

By rule 3 on Square Roots, place the hair line at 56 on the *B* scale, and on the *Reciprocal* scale read off 1337. The sum for the decimal point is

$$\sqrt{.0056} \text{ is } -1 \text{ (rule 4 on Square Roots);}$$

therefore  $1 \div \sqrt{.0056} = +1 - (-1) = +2$ , or 13.37

#### EXERCISES ON $\frac{1}{\sqrt{n}}$

Exercise 44.  $\frac{1}{\sqrt{85}}$       45.  $\frac{1}{\sqrt{7.76}}$       46.  $\frac{1}{\sqrt{.024}}$

In finding the value of  $\frac{1}{n^3}$  and also  $\frac{1}{\sqrt[3]{n}}$ , care must be taken to have the slide so that numbers on the *C* and *D* scales coincide.

To obtain the value  $\frac{1}{n^3}$ , it is only necessary to place the hair line over  $n$  on the *Reciprocal* scale and read off the answer on the *Cu* scale.

As a simple illustration  $\frac{1}{2^3} = .125$

Set the hair line over 2 on the *Reciprocal* scale and read off .125 on the *Cu* scale.

The correct placing of the decimal point in the above illustration requires, as is often the case, only the use of mental arithmetic. However, the following set of rules can be applied.

1. Ascertain how far either way the decimal point will need to be moved to accommodate the number on the *Reciprocal* scale, if it has to be moved to the left give it a plus value of three times the number of places it had to be moved, and if it has to be moved to the right give it a minus value of three times the number of places.

Thus	4	becomes	$\pm 0$
	40	"	+3
	400	"	+6
	.4	"	-3
	.04	"	-6 and so on.

2. Subtract this value from +1, because in the forming of the reciprocal value it becomes  $\frac{1}{x}$

3. If the answer falls in the first section of the *Cu* scale add -3, if in the second section add -2, or if in the third section add -1.

EXAMPLE :  $\frac{1}{2.5^3} = .064$

Setting the hair line over 2.5 on the *Reciprocal* scale, read off 64 on the *Cu* scale. It is unnecessary to move the decimal point at all, therefore it will have a value of  $\pm 0$ . Subtract this from +1 as rule 2, and, of course, the result is +1 and as the answer falls in the second section of the *Cu* scale, add -2 to this +1.

$$\begin{array}{r} \text{Thus} \quad +1 \\ \text{Add} \quad -2 \\ \hline = -1 \text{ or } .064 \end{array}$$

EXAMPLE :  $\frac{1}{29^3} = .000041$

Setting the hair line over 2.9 on the *Reciprocal* scale, read off 41 on the *Cu* scale. It is necessary to move the decimal one place to the left, so multiply this by three and commence with a value of +3. Then by rule 2,  $+1 - (+3) = -2$ . As the answer falls in the middle section of the *Cu* scale add -2.

$$\begin{array}{r} \text{Thus} \quad -2 \\ \text{Add} \quad -2 \\ \hline = -4 \text{ or } .000041 \end{array}$$

EXERCISES ON  $\frac{1}{n^3}$

- Exercise 47.  $\frac{1}{48.6^3}$       48.  $\frac{1}{.058^3}$       49.  $\frac{1}{5.75^3}$

The value  $\frac{1}{\sqrt[3]{n}}$  is found by setting the hair line over  $n$  on the *Cu* scale, and the corresponding value on the *Reciprocal* scale is the answer.

As a simple illustration  $\frac{1}{\sqrt[3]{64}} = .25$

Again the problem arises of locating the decimal point correctly. However, exactly the same rules apply as did for the taking of ordinary cube roots, with the addition that the decimal value has lastly to be subtracted from +1 because the reciprocal is being obtained.

EXAMPLE :  $\frac{1}{\sqrt[3]{98.5}} = .2164$

By rule 3 on Cube Roots (page 22), place the hair line over 98.5 in the middle section of the *Cu* scale, and on the *Reciprocal* scale read off the value .2164 (Fig. 10).

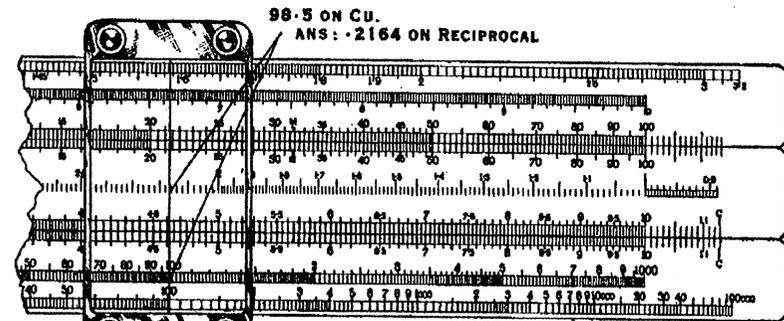


FIG. 10.

EXAMPLE :  $\frac{1}{\sqrt[3]{.775}} = 1.09$

By rule 4 on Cube Roots (page 22), use the right-hand section of the *Cu* scale, and solve by the same method as the example above to arrive at the answer of 1.09.

EXERCISES ON  $\frac{1}{\sqrt[3]{n}}$

- Exercise 50.  $\frac{1}{\sqrt[3]{870}}$       51.  $\frac{1}{\sqrt[3]{585000}}$       52.  $\frac{1}{\sqrt[3]{678}}$

## CHAPTER 9.

### LOGARITHMS

The next scale to consider is an evenly divided one the same length as, and used in conjunction with, the *D* scale to determine the logarithm of any number.

On some rules this *L* scale, as it is usually marked, is on the back of the slide, in which case to find a logarithm, the 1 on the *C* scale is set to correspond with the number on the *D* scale of which the logarithm is required, and then turning the rule over, the value is read off on the *L* scale opposite, or under, an indicating line near the end of the stock.

However, a far more convenient arrangement is when this *L* scale is on the face of the rule, it being only necessary to bring the hair line over the number on the *D* scale and read off the logarithm under the hair line on the *L* scale. This scale gives only the mantissa of the logarithm, the characteristic being determined in the usual way.

Thus to find the logarithm of 54.8, set the hair line to 548 on the *D* scale and on the *L* scale read off 739. The logarithm is thus 1.739. (Fig. 11.)

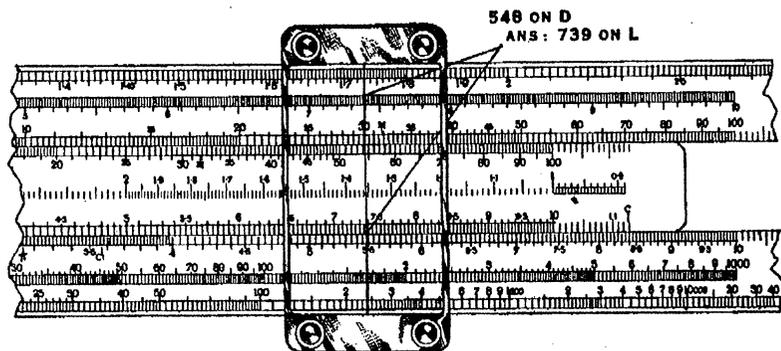


FIG. 11.

It will be seen that to find the mantissa, the decimal point

in the original number is ignored as when tables are being used. Should antilogarithms be required, the procedure is to read from the *L* scale to the *D* scale.

### NAPERIAN LOGARITHMS

(a) Using the *L* scale.

If logarithms are required to the base *e* and not to the common base 10, the common logarithm is first found and is then multiplied by a constant having the value 2.3026 (the naperian logarithm of 10).

EXAMPLE :  $\log_e 121 = 4.8$

By using the *L* scale it is found that

$$\begin{aligned} \log_{10} 121 &= 2.083 \\ \text{therefore } \log_e 121 &= 2.083 \times 2.3026 \\ &= 4.8 \end{aligned}$$

(b) Using the *Log-Log* scales.

These scales are divided into two sections, one of which is engraved along the upper edge, and the other along the lower edge of the rule.

The upper one is usually graduated from 1.1 to 3.2 and the lower from 2.5 to 100,000. Hereunder reference will be made to the *Upper* and *Lower Log-Log* scales as the *ULL* and *LLL* scales.

The number if over 2.718 is set by the hair line on the *LLL* scale and its naperian logarithm read off the *D* scale under the hair line.

If the number is less than 2.718, it is set on the *ULL* scale, and the reading on the *D* scale is divided by 10.

EXAMPLES :  $\log_e 121 = 4.8$   
 $\log_e 1.2 = .1823$

### EXERCISES ON LOGARITHMS.

Exercise 53.  $\log_{10} 871$     54.  $\log_{10} 47000$     55.  $\log_e 1.84$

CHAPTER 10.

INVOLUTION AND EVOLUTION

INVOLUTION

A slide rule with a *Log* scale can be used to find the value of  $x^a$  in the following way.

Using logarithms and calling the answer  $z$ .

$$\begin{aligned} \log x^a &= \log x \times a \\ &= \log z \end{aligned}$$

$$\text{Thus } z = \text{antilog} (\log x \times a)$$

Find the logarithm on the *L* scale of  $x$  on the *D* scale, and transfer this, with the correct characteristic to the *D* scale. Multiply by  $a$  on the *C* scale and read the product on the *D* scale. Set the mantissa of this on the *L* scale and find the antilogarithm on the *D* scale. The characteristic then determines the location of the decimal point.

EXAMPLE :  $3^{4.5} = 140.3$   
 $\log 3 = .477$  (characteristic is 0).  
 $0.477 \times 4.5 = 2.147$   
 $\text{antilog } 2.147 = 140.3$

This answer can be obtained at one setting of the rule if *Log-Log* scales are provided, in the following manner.

To find the value  $x^a$ , set  $x$  on the *ULL* or *LLL* scales as the case may be, and multiply it by  $a$  on the *C* scale, reading off the answer on the *ULL* or *LLL* scale.

EXAMPLE :  $3^{4.5} = 140.3$

Set the hair line over 3 on the *LLL* scale and bring 1 on the *C* scale to correspond with it. Then move the hair line

along to 4.5 on the *C* scale, and the corresponding point on the *LLL* scale is the answer, 140.3 (Fig. 12.)

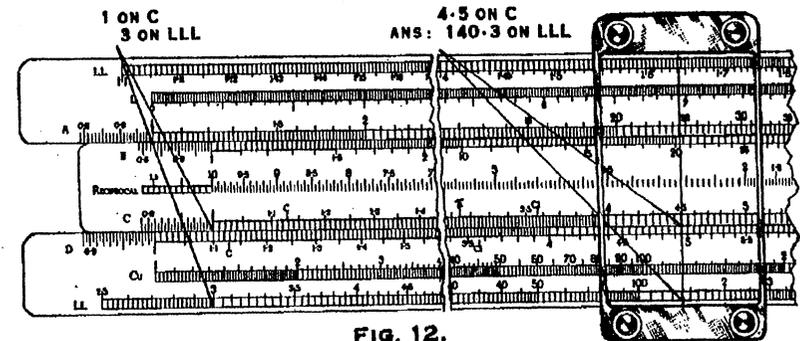


FIG. 12.

EXERCISES ON INVOLUTION.

- Exercise 56.  $1 \cdot 24^4$                       57.  $2 \cdot 67^{.05}$

EVOLUTION

To find the value  $\sqrt[a]{x}$  using the *L* scale, the logarithm of  $x$  is first found on the *L* scale and transferred with the correct characteristic to the *D* scale. It is divided by  $a$  on the *C* scale and the quotient is read on the *D* scale. The mantissa of this is set on the *L* scale and the antilog is found on the *D* scale. The characteristic then determines the location of the decimal point.

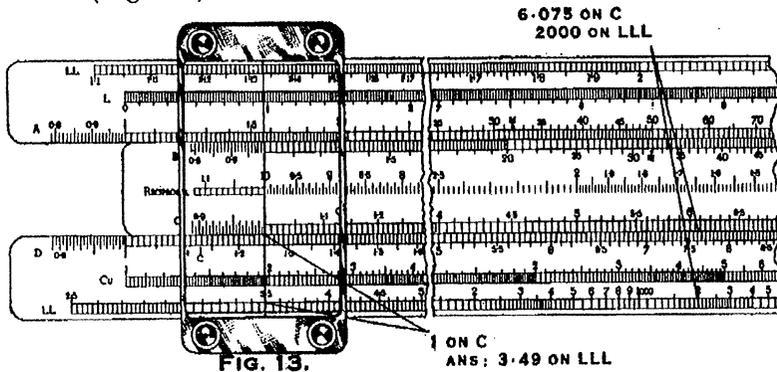
EXAMPLE :  $6.075 \sqrt[6]{2000} = 3.49$   
 $\log 2000 = .301$  (Characteristic 3)  
 $\frac{3.301}{6.075} = .543$   
 $\text{antilog of } 0.543 = 3.49$

Using the *Log Log* scales to find  $\sqrt[a]{x}$ , set  $x$  on the *ULL* or *LLL* scales, as the case may be, and divide by  $a$  on the

C scale, reading off the answer on the *ULL* or *LLL* scale.

EXAMPLE :  $6.075 \sqrt[9]{2000} = 3.49$

Set 6075 on the *C* scale over 2000 on the *LLL* scale by means of the hair line and then transferring the hair line to 1 on the *C* scale ascertain the answer on the *LLL* scale to be 3.49. (Fig. 13.).



It will be noted that when the scales are graduated between the limits 1.1 and 100,000, and it is required to work on a problem outside these limits, it is necessary to divide it into smaller factors and work them out in turn thus

$$\begin{aligned} 15^9 &= 1.5^9 \times 10^9 \\ &= 38.4 \times 10^9 \\ &= 38,400,000,000 \end{aligned}$$

$$\begin{aligned} \sqrt[9]{120000} &= \sqrt[9]{120} \times \sqrt[9]{1000} \\ &= 1.702 \times 2.154 \\ &= 3.666 \end{aligned}$$

NOTE : It will be seen that numbers on the *ULL* scale are the tenth roots of numbers on the *LLL* scale. Thus tenth roots and tenth powers can be obtained with a single setting of the cursor.

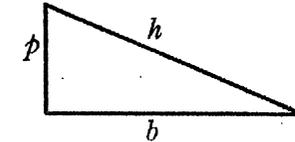
#### EXERCISES ON EVOLUTION.

Exercise 58.  $4.95 \sqrt[4]{1800}$       59.  $3.75 \sqrt[3]{16}$

## CHAPTER 11.

### TRIGONOMETRICAL PROBLEMS

Using geometrical relationships, the length of the third side of a right-angled triangle may be found from the other two using the *A*, *B*, *C* and *D* scales only and without a knowledge of trigonometry.



In the triangle illustrated  $h^2 = p^2 + b^2$

Therefore

$$h = \sqrt{b^2 + p^2}$$

$$b = \sqrt{h^2 - p^2}$$

$$p = \sqrt{h^2 - b^2}$$

Expressed in a form suitable for slide rule use, the formulae would be

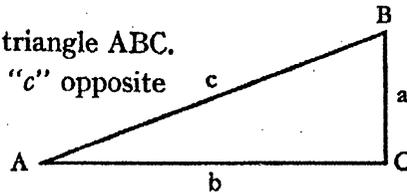
$$h = p \sqrt{\frac{b^2}{p^2} + 1} \quad b = p \sqrt{\frac{h^2}{p^2} - 1} \quad p = b \sqrt{\frac{h^2}{b^2} - 1}$$

EXAMPLE : If  $p = 25$ ,  $b = 52$ , then  $h = 57.7$

Correspond 52 on the *D* scale with 25 on the *C* scale by means of the hair line, then transfer the hair line to 1 on the *B* scale and note the value on the *A* scale. To this value, 4.33, add 1 by bringing the hair line to 5.33 on the *A* scale, bring 1 on the *B* scale under the hair line, then corresponding to 25 on the *C* scale read off the length  $h$  as 57.7 on the *D* scale.

A great majority of trigonometrical problems including the solution of triangles can be solved with a knowledge only of the six simple trigonometrical ratios which may be acquired quickly by anyone unacquainted with them.

Consider any right-angled triangle ABC.  
 If the angle C is 90° the side "c" opposite to it is known as the "hypotenuse".



The side "a" is said to be opposite to angle A and adjacent to angle B.

The side "b" is said to be opposite to angle B and adjacent to angle A.

The ratio  $\frac{\text{opposite side}}{\text{hypotenuse}}$  is known as the "Sine" or "Sin" of the angle, thus  $\frac{a}{c} = \text{Sin A}$ , and  $\frac{b}{c} = \text{Sin B}$ .

The ratio  $\frac{\text{adjacent side}}{\text{hypotenuse}}$  is known as the "Cosine" or "Cos" of the angle, thus  $\frac{a}{c} = \text{Cos B}$ , and  $\frac{b}{c} = \text{Cos A}$ .

It will be noted in the above examples that  $\text{Sin A} = \frac{a}{c} = \text{Cos B}$ , and  $\text{Cos A} = \frac{b}{c} = \text{Sin B}$ . Now  $A+B+C = 180^\circ$  and  $C = 90^\circ$ , therefore  $A+B = 90^\circ$  and thus  $A = 90^\circ - B$  and  $B = 90^\circ - A$ .

It follows then that  $\text{Sin A} = \text{Cos B} = \text{Cos}(90^\circ - A)$ , and  $\text{Cos A} = \text{Sin B} = \text{Sin}(90^\circ - A)$ .

Advantage is taken of this fact and on slide rules the one scale is used to give both Sines and Cosines.

The ratio  $\frac{\text{opposite side}}{\text{adjacent side}}$  is known as the "Tangent" or "Tan" of the angle, thus  $\frac{a}{b} = \text{Tan A}$ , and  $\frac{b}{a} = \text{Tan B}$ .

Then there are reciprocal functions of the ones given above. The reciprocal of the Sin is known as the "Cosecant" or "Cosec" and is given by  $\frac{\text{hypotenuse}}{\text{opposite side}}$ . The reciprocal of the

Cosine is known as the "Secant" or "Sec" of the angle and is given by  $\frac{\text{hypotenuse}}{\text{adjacent side}}$ .

Thus  $\frac{c}{a} = \text{Cosec A} = \text{Sec B}$ , and  $\frac{c}{b} = \text{Cosec B} = \text{Sec A}$ .

The reciprocal of the Tangent is known as the "Cotangent" or "Cot" and its value is given by  $\frac{\text{adjacent side}}{\text{opposite side}}$ .

Thus  $\frac{b}{a} = \text{Cot A} = \text{Tan B}$  (from above)

$\frac{a}{b} = \text{Cot B} = \text{Tan A}$  (from above)

In a manner similar to that for Sine and Cosine it can be shown that  $\text{Tan A} = \text{Cot}(90^\circ - A)$ , and  $\text{Cot A} = \text{Tan}(90^\circ - A)$ , and on slide rules the one scale may be used to give both Tangent and Cotangent values.

It will be noted that all of the above are ratios and quite independent of the actual dimensions of the triangle, yet constant for any one angle.

#### TRIGONOMETRICAL RATIOS ON SLIDE RULES.

The scales of trigonometrical ratios on slide rules are of two types, non-logarithmic and logarithmic.

The non-logarithmic scales of Sin and Tan values are engraved adjacent to scales of equivalent angles and may be read off by eye or by using the hair-line to judge co-incidence. (Fig. 14.)

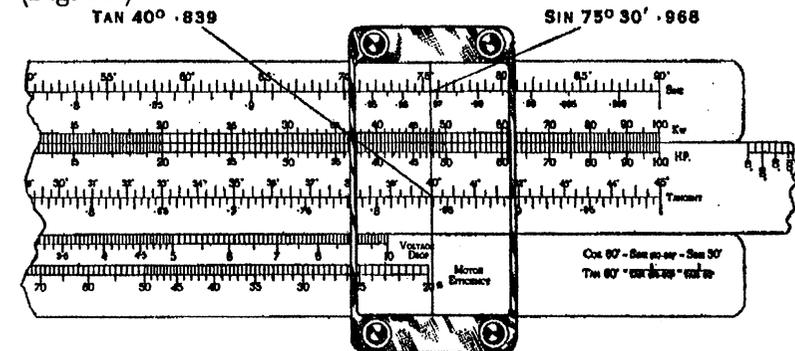


FIG. 14.

This type of scale has the advantage of greater accuracy with the larger angles where both the ratio and its logarithm vary slowly. However, where the reciprocal functions are required or multiples needed the numerical value of the function must be transferred to the *C* scale and the reciprocal read off on the *Reciprocal* scale or multiplication carried out in conjunction with the *D* scale.

Logarithmic scales of trigonometrical functions are variously placed on the stock or on either side of the slide. They may consist of half-size logarithmic units for use with the *A* and *B* scales or full sized units for use with the *C* and *D* scales. Fundamentally, the majority of these scales are identical although differences in size, and location on the rule, necessitate differences in manipulation.

On the "Manheim" type of slide rule they are found on the reverse side of the slide. Along the upper edge is the *Sine* scale graduated from  $0^\circ 34'$  to  $90^\circ$ , and used in conjunction with the *A* and *B* scales on the face of the rule. Along the lower edge is the *Tangent* scale graduated from  $5^\circ 43'$  to  $45^\circ$  and used in conjunction with the *C* and *D* scales. Where tangents of angles between  $0^\circ 34'$  and  $5^\circ 43'$  are required, the corresponding sine value, which is approximately equal, must be used.

EXAMPLE :  $\sin 45^\circ = .707$ .

Turn the rule over and withdraw the slide until the  $45^\circ$  on the *Sine* scale corresponds with the indicating line at the right-hand end of the rule, then turning the rule over again to the face, read off on the *B* scale, at the point opposite the right-hand end of the *A* scale 70.7.

To find the cosine of an angle, subtract the angle from  $90^\circ$  and ascertain the sine thereof because  $\text{Cosine } \theta = \text{Sine } (90^\circ - \theta)$ .

EXAMPLE :  $\tan 25^\circ = .466$ .

Turn the rule over and withdraw the slide until  $25^\circ$  on the *Tangent* scale corresponds with the indicating line at the left-hand end of the rule, then, turning the rule over again to the face, read off on the *C* scale opposite the 1 on the *D* scale 4.66.

Cotangent  $25^\circ$  is the reciprocal of tangent  $25^\circ$ , and this value, 2.145, will be found on the *D* scale opposite the 10 on the *C* scale.

It will be observed that the *Tangent* scale is graduated to only  $45^\circ$ , so when tangents or cotangents of angles above  $45^\circ$  are required, the following procedure is to be adopted.

To find the value tangent  $\theta$  when  $\theta$  is greater than  $45^\circ$ , take the cotangent of  $(90^\circ - \theta)$ .

To find the value cotangent  $\theta$  when  $\theta$  is greater than  $45^\circ$ , take the tangent of  $(90^\circ - \theta)$ .

On the "Reitz" pattern rule, the *Tangent* scale on the lower edge of the reverse side of the slide ranges from  $5^\circ 43'$  to  $45^\circ$  as on the "Manheim" type but the *Sine* scale on the upper edge is graduated from  $5^\circ 44'$  to  $90^\circ$ . A third scale centrally placed and marked *S* & *T* gives sines and tangents (considered identical) of angles from  $0^\circ 34'$  to  $5^\circ 44'$ . All of these scales are used in conjunction with the *C* and *D* scales on the face of the rule, and apart from this difference the manipulation is the same as for the "Manheim" type.

When the *Sine* and *Tangent* scales are found on the face of the rule they are similar in graduation layout to those found on the reverse side of the slide, but their proximity to the standard *A*, *B*, *C* and *D* scales enables the numerical values of the functions to be read using the cursor alone and without a special setting of the slide.

Where both the *Sine* and *Tangent* scales are used in conjunction with the *C* and *D* scales, they sometimes consist of two full sized logarithmic units each. An upper scale ranging from  $30'$  to  $6^\circ$  and a lower scale from  $5^\circ$  to  $90^\circ$  for sines, while for tangents there is a similar but separate upper scale and a lower scale ranging from  $5^\circ$  to  $45^\circ$ . On each of these four scales the complements of the angles are engraved in red to facilitate their use in obtaining cosines and cotangents.

With this type of rule, when the cursor is brought to an angle on the *Sine* or *Tangent* scale it automatically co-incides with the numerical value of the function on the *C* or *D* scale respectively. From this stage, multiples, reciprocals, powers, etc. may be found by ordinary arithmetical methods outlined in

earlier chapters. Where the trigonometrical scales are designed for use with the *C* and *D* scales, the squares of the functions may be obtained by using the *A* and *B* scales instead. For a practical application of this, see the paragraph on stadia reduction.

EXAMPLE :  $11.1 \sin 8^\circ 55' = 1.72$ .

Set *C* index to 11.1 on the *D* scale and the cursor to  $8^\circ 55'$  on the *Sine* scale. On the *C* scale under the cursor note .155 which is  $\sin 8^\circ 55'$  and on the *D* scale read off  $1.72 = 11.1 \sin 8^\circ 55'$ . (Fig. 15.)

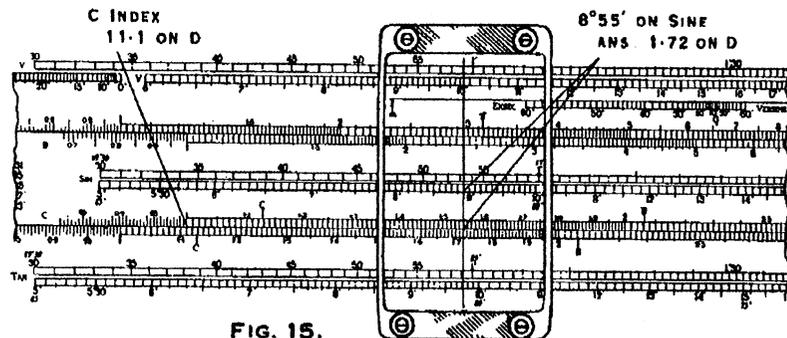


FIG. 15.

A careful study of the rule set in this position will reveal the procedure to be adopted in the solution of the following problems.

$$123 \sin^2 8^\circ 55' = 123 \times .024 = 2.96$$

$$\frac{155}{1720} = .0901 = \sin 5^\circ 10'$$

$$\frac{155}{17200} = .00901 = \sin 0^\circ 31'$$

$$111 \cos 81^\circ 05' = 111 \times .155 = 17.2$$

$$11.1 \sin 0^\circ 53' 20'' = 11.1 \times .0155 = .172$$

$$\tan 9^\circ 46' \div 155 = \cot 80^\circ 14' \div 155 = \frac{0.172}{155} = .00111$$

$$\frac{172}{1550} = .111 = \tan 6^\circ 20' = \cot 83^\circ 40'$$

$$\frac{1.72}{155} = .0111 = \tan 0^\circ 38' 10''$$

$$\tan^2 9^\circ 46' \div 24 = \frac{.0296}{24} = .00123$$

On certain rules modified for artillery use the two *Sine* scales are designated *Angle 1* and *Angle 2*, while the *D* scale used in conjunction with them is termed the *Range* scale. The mode of operation is very similar to the foregoing.

In addition to the normal trigonometrical functions there are several other trigonometrical scales which are particularly useful. Thus the functions external secant or exsec, i.e. (secant-1) and versine, i.e. (1-cosine) take the place of secant and cosine in calculations where the angle is small and the accuracy required is greater than that given by the scales of the normal functions. The *Exsec* and *Versine* scales are of the auxiliary type and are situated on the stock of the rule. The angle in degrees and decimals thereof is located on the *C* scale and brought into co-incidence with the corresponding graduation on the auxiliary scale and the numerical value of the function is found on the *B* scale under the central index at 10 on the *A* scale. The decimal point is fixed by consulting the table of versines on the slide. This table is also approximately correct for external secants. The slide is thus set so that multiples of the function may be obtained by moving the cursor alone, and the scales and index so situated that re-settings of the slide are reduced to a minimum. A constant mark "m" to the left of the main scale is the "0" graduation repeated together with adjacent graduations in such a position that for small angles it can be used in conjunction with the number on the *C* scale representing the angle in minutes instead of degrees and decimals thereof.

EXAMPLE :  $\cos 11^\circ 27' = 1 - \text{Versine } 11^\circ 27' = 1 - 0.199 = .801$

Bring the cursor to  $11^\circ 27'$  on the *Versine* scale and 11.45 on the *C* scale to the cursor. On the *B* scale under the central index read .0199 = *Versine*  $11^\circ 27'$ .  $\cos 11^\circ 27' = 1 - 0.199 = .801$ .

EXAMPLE :  $66 \text{ Secant } 16^\circ 36' = 68.87$ .

This can be stated as  $66 + 66 \text{ Exsec } 16^\circ 36'$ .

Bring into co-incidence  $16.6^\circ$  on the *Exsec* scale with 16.6 on the *C* scale. On the *B* scale under the central index note .0435 which is  $\text{Exsec } 16^\circ 36'$  and on the *B* scale under *A* 66 read off 2.87.

Therefore  $66 \text{ Secant } 16^\circ 36' = 66 + 2.87 = 68.87$ .

EXAMPLE: Find the horizontal equivalent of 275.028 feet measured along a slope of  $2^\circ 03'$ .

This can be stated as  $275.028 \text{ Cos } 2^\circ 03'$  or  $275.028 - 275.028 \text{ Versine } 2^\circ 03'$ .

By means of the cursor bring *C* 123 ( $2^\circ 03' = 123'$ ) to correspond with "m" on the *Versine* scale and on the *B* scale under the central index note .00064 which is  $\text{Versine } 2^\circ 03'$ . On the *B* scale under *A* 275 read off .176. Therefore the required length is  $275.028 - 0.176 = 274.852$  feet.

### ARCS AND CHORDS

*Arc* and *Chord* correction scales are similar in principle and operation to the *Exsec* and *Versine* scales. The *Arc* scale is used to obtain the correction to be added to the chord to ascertain the arc, the subtended angle being known. Similarly the *Chord* scale is used to obtain the correction to be subtracted from a known arc to give the corresponding chord. The subtended angle will generally be known, but should the radius of the curve be given instead, the angle must first be found by separate calculation. When this angle is less than  $10^\circ$ , the constant "M" situated to the left of the main scales may be used in conjunction with the angle in minutes on the *C* scale.

To find the length of an arc when the chord and subtended angle are given, the following procedure is adopted.

By means of the cursor bring the angle in degrees and decimals thereof on the *C* scale under the corresponding graduation on the *Arc* scale. On the *B* scale under the central index read the correction expressed as a decimal of the chord which, for the purpose of fixing the decimal point, may be reckoned as approximately one tenth of the versine of the angle. Move

the cursor to the chord length on the *A* scale and on the *B* scale at the cursor read the length to be added to the chord to give the arc. A re-setting of the slide may be avoided by using *A* 100 as the index but care must be taken to correctly locate the decimal point.

EXAMPLE: Find the length of arc when the chord equals 48.48 feet and the subtended angle is  $70^\circ$ .

By means of the cursor set *C* 70 under  $70^\circ$  on the *Arc* scale. On the *B* scale under the central index note 6.50, the correction to a 100 foot chord. Move the cursor to *A* 48.48 and on the *B* scale read off 3.15. Therefore the arc =  $48.48 + 3.15 = 51.63$  feet.

To find the chord when the length of arc is given, the procedure is similar to that of above except that the *Chord* scale is used and the correction is subtracted.

EXAMPLE: Find the chord of an arc 306.39 feet in length which has a subtended angle of  $35^\circ 24'$ .

Using the cursor bring *C* 35.4 to correspond with  $35^\circ$  on the *Chord* scale. On the *B* scale under the central index note 1.58 which is the correction for a 100 foot arc. On the *B* scale under *A* 306 read off 4.85. Therefore the length of the chord =  $306.39 - 4.85 = 301.54$  feet.

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### STADIA SCALES

These scales are so useful to surveyors and engineers that slide rules are frequently manufactured for stadia reduction only. However they are also incorporated in some general purpose trigonometrical rules.

The *H* scale is a logarithmic scale of values of  $\text{Cos}^2 \theta$  and the *V* scale of  $\frac{1}{2} \text{Sin } 2 \theta$ . According to the design of the rule, they are used in conjunction with either the *A* and *B* scales or the *C* and *D* scales, in the solution of the formulae  $h = G \text{ Cos}^2 \theta$  and  $v = G \frac{1}{2} \text{Sin } 2 \theta$  where the generator "G" (staff reading  $\times$  constant) and the angle  $\theta$  are known, and the

horizontal distance "h" and the difference in elevation "v" are required. Some stadia slide rules have an *H C* scale of values of  $\sin^2 \theta$  so that  $1 - \sin^2 \theta$  may be used instead of  $\cos^2 \theta$  in the formula when the angle is small. However, on trigonometrical rules where the *Sine* scale accords with the *C* scale, values of  $\sin^2 \theta$  are found on the *B* scale.

EXAMPLE: Given  $G = 169$  and  $\theta = 8^\circ 15'$ , find "v" and "h".

Bring *C* 169 to the *D* index, then set the cursor to  $8^\circ 15'$  on the *V* scale. On the *D* scale under the cursor note .142 (the numerical value of  $\frac{1}{2} \sin 2\theta$ ) and on the *C* scale read off "v" as 24.0. (Fig. 16.)

Then move the cursor to  $8^\circ 15'$  on the *H* scale. On the *D* scale under the cursor note 0.98 (the numerical value of  $\cos^2 \theta$ ) and on the *C* scale read off "h" as 165.5.

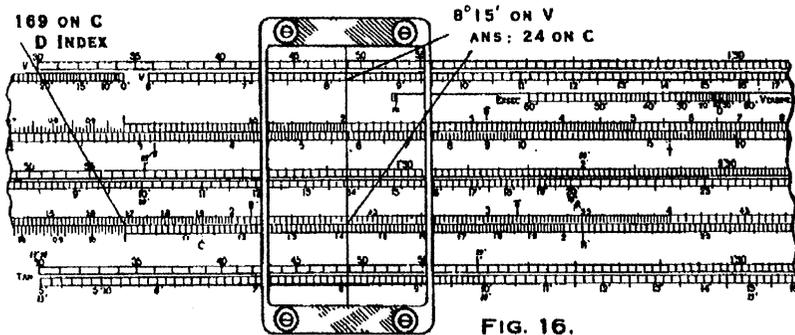


FIG. 16.

### CIRCULAR MEASURE

When the sine or tangent of an angle smaller than  $3^\circ$  is required, you may without error use the radian measure of the angle. As this is proportional to the magnitude of the angle in degrees etc., no special scales are necessary, and the required value may be obtained by the use of constants found

on the *C* and *D* scales at 57.3, 3438 and 206265. These numbers are respectively the number of degrees, minutes and seconds in one radian and the constant marks may be designated  $R^\circ$ ,  $R'$ ,  $R''$ , or in Greek letters or by some other symbols. Circular measure is also used in other problems such as those dealing with curves, etc.

EXAMPLE: Find  $1^\circ 45'$  in radians.

Noting that  $1^\circ 45'$  is  $1.75^\circ$  or 105', set  $R^\circ$  on the *C* scale to 1.75 on the *D* scale ( $R'$  automatically co-inciding with 105) and on the *D* scale below *C* index read off .03054 which is the required result.

EXAMPLE: Find the deviation of a bearing for an offset of 2.5 links at a distance of 4150 links.

This can be stated as:— Find the angle the tangent or radian measure of which is represented by the fraction  $\frac{2.5}{4150}$ .

Set 4150 on the *C* scale to 2.5 on the *D* scale and on the *D* scale below  $R'$  read 124" which is  $2' 04''$ .

EXAMPLE: Find the length of a curve of 8 chains radius with a subtended angle of  $59^\circ$ .

Set  $R^\circ$  to *D* 59 and on the *D* scale under *C* index note 1.03 which is  $59^\circ$  in radians. Move cursor to *C* 8 and on the *D* scale read off the length of the curve as 824 links.

### SOLUTION OF TRIANGLES

Many trigonometrical problems can be resolved simply into the solution of a triangle or a series of triangles, and a vast majority of triangle solutions are covered by the "Sine Rule", the operation of which on a slide rule is a matter of extreme simplicity.

The Sine Rule covers the following cases:—

- (a) Where two angles and a side are given. The third angle is found by subtracting the sum of the other two from  $180^\circ$ .
- (b) Where two sides and a non-included angle are known. (If the given angle is less than  $90^\circ$  and is opposite the smaller of the two sides there are two solutions.)

(c) Any right-angle triangle except the one case where the sides forming the right angle are the only known factors. All right-angled triangles, including the exception quoted, can also be solved by using the six simple trigonometrical ratios but it is convenient in slide rule work to use the "Sine Rule" and so standardize procedure.

The Sine Rule formula is  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ .

In any triangle in which a side and an angle opposite to it are known, if the angle on the *Sine* scale is brought into co-incidence with the length of the side on the appropriate numerical scale, then the remaining two angles co-incide with the sides opposite to them.

For purposes of comparison of methods the triangle ABC in fig. 17 will be used in all examples. Any figures which cannot be handled by the method in question will be rounded off.

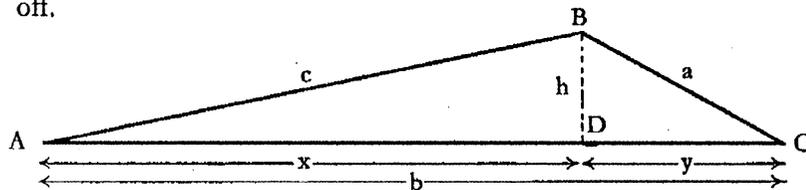


FIG. 17.

A = 10° 40'	a = 204.154	h = 91.624
B = 142° 40'	b = 668.896	x = 486.457
C = 26° 40'	c = 495.011	y = 182.439

EXAMPLE: Solve triangle ABC given A = 10° 40', C = 26° 40', c = 495.

Therefore B = 180° - A - C = 142° 40'.

As Sines of angles greater than 90° are not generally shown on slide rules, use is made of the fact that  $\sin B = \sin 180^\circ - B$ .

Thus  $\sin B = \sin (A + C) = \sin 37^\circ 20'$ .

Set 26° 40' on the *Sine* scale to 495 on the appropriate numerical scale (either A or D according to the type of rule). Co-inciding with  $\sin 10^\circ 40'$  read 204 = "a" and with  $\sin 37^\circ 20'$  read 669 = "b".

EXAMPLE: Solve triangle ABC given A = 10° 40', a = 204, c = 495.

Set 10° 40' on the *Sine* scale to 204 on the numerical scale. On the *Sine* scale co-inciding with 495 on the numerical scale read off 26° 40' = C.

Therefore B = 180° - 10° 40' - 26° 40' = 142° 40', the Sin of which has the same value as  $\sin 37^\circ 20'$ .

Co-inciding with  $\sin 37^\circ 20'$  read 669 = b.

However  $\sin 26^\circ 40'$  has the same value as  $\sin 153^\circ 20'$ , giving an alternative value for C. In this case B will be  $180^\circ - 10^\circ 40' - 153^\circ 20' = 16^\circ 0'$ , and on the numerical scale co-inciding with  $\sin 16^\circ$  read 304 = b.

EXAMPLE: Solve triangle ABD given D = 90°, A = 10° 40' and x = 486 (the notation is as in fig. 17).

The angle B in this triangle will be  $180^\circ - 90^\circ - 10^\circ 40' = 79^\circ 20'$ .

Set 79° 20' on the *Sine* scale to 486 on the numerical scale.

Co-inciding with  $\sin 90^\circ$  read 495 = c.

Co-inciding with  $\sin 10^\circ 40'$  read 91.6 = h.

Two cases arise in which preliminary calculations are required to find an additional angle before the Sine Rule can be applied.

1. Where two sides and an included angle are given, there are two methods available:—

(a) Given "b", "c" and the angle A, use the slide rule to find the magnitude of the angle  $\frac{B-C}{2}$  in the formula  $\tan \frac{B-C}{2} = \frac{b-c}{b+c} \tan \frac{B+C}{2}$  where  $\frac{B+C}{2} = \frac{180^\circ - A}{2}$ . Knowing  $\frac{B+C}{2}$  and having found  $\frac{B-C}{2}$  the sum of the two angles is angle B and the difference is angle C. Apply the Sine Rule to find the remaining side.

EXAMPLE: Solve triangle ABC given A = 10° 40', c = 495, b = 669.

$$\begin{aligned} \tan \frac{B-C}{2} &= \frac{669-495}{669+495} \quad \tan \frac{180^\circ-10^\circ 40'}{2} \\ &= \frac{174}{1164} \tan 84^\circ 40' \\ &= 1.60 \end{aligned}$$

$$\text{therefore } \frac{B-C}{2} = 58^\circ 0'$$

$$B = 84^\circ 40' + 58^\circ 0' = 142^\circ 40'$$

$$C = 84^\circ 40' - 58^\circ 0' = 26^\circ 40'$$

Thence find the remaining sides by the Sine Rule as outlined above.

(b) Divide the triangle into two right angled triangles by dropping a perpendicular from the extremity of the shorter side onto the longer. (Fig. 17.)

$$\text{Then } h = c \sin A = 495 \sin 10^\circ 40' = 91.6$$

$$x = c \cos A = 495 \cos 10^\circ 40' = 486$$

$$y = b - x = 669 - 486 = 183$$

$$\frac{h}{y} = \frac{91.6}{183} = .501 = \tan C.$$

$$\text{Therefore } C = 26^\circ 38'.$$

Complete the triangle by applying the Sine Rule.

On trigonometrical rules where the *Exsec* and *Versine* scales are available, the same method may be used and greater accuracy obtained.

$$h = c \sin A = 495 \sin 10^\circ 40' = 91.6$$

$$x = c - (c \text{ Versine } A) = 495.01 - 8.55 = 486.46$$

$$y = b - x = 668.90 - 486.46 = 182.44$$

$$\frac{h}{y} = \tan C \quad \frac{h}{y} = \frac{91.6}{182.4} = .502 = \tan 26^\circ 40'$$

$$a = y + (y \text{ Exsec } C) = 182.44 + 21.7 = 204.1$$

$B = 180^\circ - A - C$ . Where the accuracy required warrants additional manipulation this latter method can be modified and used instead of the Sine Rule.

2. Where three sides only are known.

In this case one of the "half-angle" formulae must be used.

The most convenient is:—

$$\tan \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \text{ where } s = \frac{a+b+c}{2}.$$

By using "a" to denote the smallest side (as in Fig. 17) "A" must be less than  $45^\circ$  and the value of its tangent can

be read direct from the *D* scale if the *A* and *B* scales are used to evaluate  $\frac{(s-b)(s-c)}{s(s-a)}$ .

Care must be taken to use the left hand section of the *A* and *B* scales for numbers in the range 1 to 10, 100 to 1000, etc., and the right hand section for numbers in the range 10 to 100, 1000 to 10000 etc.

The foregoing is to ensure that the result falls in the correct section of the *A* scale so that on the *D* scale you can read off the square root which is  $\tan \frac{A}{2}$ . Angle  $\frac{A}{2}$  is then found on the *Tangent* scale and *A* deduced. The remaining angles may then be found by the Sine Rule.

On trigonometrical slide rules where the *Tangent* scale is on the stock adjacent to the *D* scale, the numerical value of  $\tan \frac{A}{2}$  need not be noted and one step is thus eliminated.

EXAMPLE:—Solve triangle ABC given  $a = 204$ ,  $b = 669$ ,  $c = 495$ .

$$a = 204$$

$$b = 669$$

$$c = 495$$

$$s - a = 480$$

$$s - b = 15$$

$$s - c = 189$$

$$\text{Sum} = 1368 = 2S$$

$$\text{Sum} = 684 = S \text{ (check)}$$

Evaluate  $\frac{15 \times 189}{684 \times 480}$  on the *A* and *B* scales, the final cursor setting being at *A*  $86.3$ .

The square root is at the cursor on *D* scale giving  $\tan \frac{A}{2} = .0929$ .

$$\text{Therefore } \frac{A}{2} = 5^\circ 19' \text{ and } A = 10^\circ 38'.$$

The remaining angles are found by applying the Sine Rule.

The error of 2' in angle "A" results from the omission of the decimal fractions.

#### EXERCISES ON TRIGONOMETRICAL PROBLEMS

60.  $186 \sin 18^\circ 10'$ .
61.  $146 \tan 13^\circ 20'$ .
62.  $15.7 \cot 23^\circ 30'$ .
63.  $227.18 \cos 2^\circ 13'$ . (Using Versine).
64.  $138 \operatorname{Exsec} 28^\circ$ .
65.  $138 \operatorname{Sec} 28^\circ$ .
66. In triangle ABC  $a = 72.0$ ,  $c = 67.5$ ,  $C = 51^\circ$ . Find A, B, and  $b$ .
67. Reduce stadia observations given  $G = 193$ ,  $\theta = 13^\circ 10'$ .

#### CONSTANT MARKS

In connection with the trigonometrical functions of a slide rule are what are known as gauge marks or constants, " $\pi$ " " $2\pi$ " " $M$ " " $C$ " and " $C^1$ ".

On some rules there appear the constants  $R^\circ$ ,  $R'$  and  $R''$ , and the use of these is outlined in the section on Circular Measure of this Manual.

$\pi$  and  $2\pi$  are, of course, very frequently used values when solving problems on circles. The  $\pi$  mark on the  $A$  scale can be made very useful in the following manner.

Set the 1 of the  $B$  scale under the  $\pi$  on the  $A$  scale, and with the rule thus set, the  $A$  and  $B$  scales form a table of ratios between circumferences and diameters, also the scales  $A$  and  $C$  form a table of ratios between areas and radii.

The constant " $M$ " on the  $A$  and  $B$  scales signifies the reciprocal of  $\pi$ , 0.318, and is useful on occasions in that it saves a setting of the slide.

Either of the constants " $C$ " or " $C^1$ " can be used for ascertaining the area of a circle when the diameter is known.

This is done by setting " $C$ " or " $C^1$ " on the  $C$  scale over the known diameter on the  $D$  scale, and on the  $A$  scale corresponding with either the 1 or 10 on the  $B$  scale, depending whether " $C$ " or " $C^1$ " was used, will be found the area required, since

$$\text{Area of Circle} = \text{Diam}^2 \times .7854$$

Passing from the  $C$  scale to the  $B$  scale squares the diameter and the " $C$ " or " $C^1$ " perform the multiplication of .7854. The object of providing these two constants to perform a similar task is that preference may be given to the one which necessitates the least movement of the slide.

## CHAPTER 12.

### ELECTRICAL PROBLEMS

To solve electrical problems the *A* and *B* scales are used to do the working, and the answers read off on what are known as the *Efficiency* and *Voltage Drop* scales, but there are instances when a setting is made on these latter scales, and the answer read off the *A* or *B* scales.

As with the trigonometrical scales, so with the electrical scales, manufacturers of slide rules arrange the *Voltage Drop* and *Efficiency* scales rather differently. One type has these scales in the trough of the rule, and settings are made with a metal indicator attached to the end of the slide, whereas another type of rule has a special pair of *A* and *B* scales, and the *Voltage Drop* scale is read by means of the hair line, and the *Efficiency* scales by means of an indicating arrow. In principle, both of these types are very similar.

It must be noted that these scales are only applicable to direct current calculations, or for alternating current that is free from induction.

### EFFICIENCY OF DYNAMOS

For this purpose the *A* scale is taken as the kilowatt scale, and the *B* scale as the horse-power scale, commencing at 10 horse-power.

Set 10 horse-power under 746 (usually marked as a constant), on the *Kw.* scale, and it will be seen that the efficiency shown by the indicator is, 100%. Thus, with the rule in this position, all ratios between the *Kw.* and *H.P.* scales represent 100% efficiency.

As the slide is moved to the left, thus giving lower kilowatt readings per horse-power, the indicator shows a decreasing efficiency.

From the foregoing it will be seen that having made a setting on the *Efficiency* scale, the dynamo output in kilowatts will be shown on the *Kw.* scale directly over the horse-power, on the *H.P.* scale, needed to drive the dynamo.

EXAMPLE : What would be the output with 25 horse-power from a dynamo of 75% efficiency ?

Set the indicator to 75% on the *Dynamo Efficiency* scale, and corresponding with 25 on the *H.P.* scale, the answer 14.0 kilowatts will be found on the *Kw.* scale. (Fig. 15.)

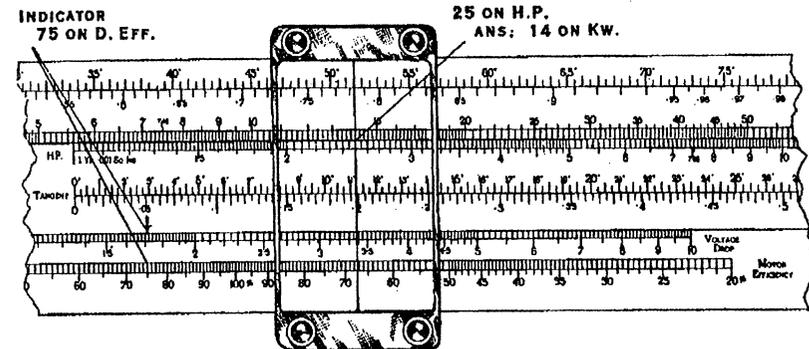


FIG. 18.

EXAMPLE : What is the efficiency of a dynamo giving 13.8 kilowatts with an input of 23 horse-power ?

Set the hair line to 13.8 on the *Kw.* scale and bring 23 on the *H.P.* scale to correspond with it, and then read off the indicator 80.5% on the *Dynamo Efficiency* scale.

EXAMPLE : What horse-power would be needed to drive a dynamo 75% efficient supplying 40 kilowatts ?

Set the indicator to 75% and on the *H.P.* scale, opposite 40 on the *Kw.* scale, read the answer as 71.5 horse-power.

### EFFICIENCY OF MOTORS

The *Motor Efficiency* scale is used in a similar manner to the *Dynamo Efficiency* scale to determine the consumption of a motor of a known horse-power. As before, the *A* and *B* scales are taken as the *Kilowatt* and *H.P.* scales.

EXAMPLE : Find the efficiency of a motor which, with 19 kilowatts, delivers 19.9 horse-power.

Line up these two values on their proper scales, and off the indicator read the answer 78% on the *Motor Efficiency* scale.

EXAMPLE : Ascertain the power in kilowatts, required by a motor developing 41 horse-power, having an efficiency of 84%.

Set the indicator to 84% on the *Motor Efficiency* scale and opposite 41 on the *H.P.* scale read the answer on the *Kw.* scale as 36.4 kilowatts.

EXAMPLE : What horse-power would a 70% efficient motor deliver with 25 kilowatts ?

Set the indicator this time to 70% on the *Motor Efficiency* scale, and opposite 25 on the *Kw.* scale read the answer on the *H.P.* scale as 23.5 horse-power.

#### VARIATION OF RESISTANCE WITH TEMPERATURE OF A COPPER CONDUCTOR

This is determined by using the small *Temperature* scale situated sometimes in the centre, and sometimes at the right-hand end of the slide.

*British Standard Specifications No. 7*, accepted by the Standards Association of Australia, determines the effect on the resistance of a copper conductor for temperatures ranging between 40° and 130° F., standard conditions being 60° F.\*

By setting 60° F. on this scale to the resistance in ohms on the *Kw.* scale, the resistance at other temperatures between 40° and 130° F. can be found. Similarly, if the resistance is known at any temperature between 40° and 130° F. its value at the standard temperature of 60° F. can be obtained.

EXAMPLE : The resistance of a copper conductor at 60° F. is 11 ohms. What is the resistance at 80° F. ?

Set 60° on the *Temp.* scale under 11 on the *Kw.* scale, and opposite 80° read off the answer of 11.5 ohms.

\*Only slide rules with temperature engravings from 40° to 130° F. conform to this specification. Other rules are either marked in °C. or in Fahrenheit equivalents of 0, 15, 25, 50 and 75° C., a range greater than that covered in *B.S.S. No. 7*.

The following remarks and those referring to the *Voltage Drop* scale will only apply to rules correct to *B.S.S. No. 7*.

EXAMPLE : What is the resistance at 60° F. of a conductor having a resistance of .5 ohms at 100° F. ?

Set 100° on the *Temp.* scale under 5 on the *Kw.* scale, then the required resistance of .459 is found opposite 60°.

The *Temperature* scale can also be used to vary the cross sectional area of a conductor whose resistance at working temperature must not exceed a certain value, when the design of a conductor has been carried out at the standard temp. of 60° F.

EXAMPLE : A conductor of .02 square inches is found necessary at a temperature of 60° F. What would be the area necessary for a working temperature of 50° F., assuming that the resistance is to be kept constant ?

As the resistance is reduced for a drop in temperature, a smaller conductor would be satisfactory.

Set 60° to 20, *i.e.*, .02 sq. ins. on the *Kw.* scale and read off 19.6, *i.e.*, .0196 sq. ins., opposite 50° on the *Temperature* scale.

#### VOLTAGE DROP

The voltage drop is equal to the current flowing in amps multiplied by the resistance of the conductor in ohms ; this resistance varies directly with the length of the conductor and inversely with the cross-sectional area.

The *Voltage Drop* scale is so constructed that the drop is given for a copper conductor having a resistance of .0240079 ohms for a length of 1000 yards and a cross-sectional area of one square inch, which is the *B.S.S.* figure given for a temperature of 60° F.

In order to simplify the calculation the current scale has been graduated from 1 to 100 to correspond with currents of from 10 to 1000 amps, the left-hand index being marked 10 amps.

The length and cross-sectional area scale has been graduated in a similar way to correspond to lengths of from 1 to 100 yards, and areas from .001 to .1 sq. inches, with the left-hand index marked 1 yd. and .001 sq. ins.

The voltage drop is obtained by the following formula :—

$$\text{Voltage Drop} = \frac{\text{Length in yards} \times \text{Current in amps} \times \text{Resistance of Copper}}{\text{Area in sq. ins.}}$$

The *Voltage Drop* scale is so placed with reference to the other scales that the multiplication by the resistance of copper in this formula is automatically performed in reading the answer on the *Voltage Drop* scale.

EXAMPLE : What is the voltage drop when 30 amps flow along a conductor 50 yards in length having a cross sectional area of .0045 sq. ins. ?

Set the hair line to 3 on the amps scale, *i.e.*, 30 amps, and bring the 4.5, *i.e.*, .0045 sq. ins. on the slide to correspond with it. Move the hair line along to 50 yards and read off the corresponding value on the *Voltage Drop* scale of 8.1 volts.

EXAMPLE : What is the voltage drop when 20 amps flow along a conductor 1200 yards in length, having a cross-sectional area of .25 sq. ins. ?

From a knowledge of the rule it is obvious that 1200 yards cannot be set on the rule, nor can .25 sq. inches ; however, as the resistance of the conductor and also the voltage drop vary directly with the length, and inversely with the cross-sectional area, a length of 12 yards and an area of .0025 sq. inches will give the same result.

Set the hair line to 20 amps and bring .0025 sq. ins. under it. Move the hair line along to 12 yards and read the result under it as 2.32 volts.

EXAMPLE : What is the voltage drop in a conductor 900 yards long with an area of .0145 sq. ins. when 2.5 amps flow along it ?

The current of 2.5 amps cannot be directly read on the rule, but as the voltage drop varies directly as the length and current flowing in the conductor, 2.5 amps and 900 yards would have the same effect as 25 amps along 90 yards. The

problem then is to find the voltage drop along the shorter length with the greater current and the same cross section.

Set the hair line to 25 amps on the amps scale and bring .0145 sq. ins. on the slide to correspond with it. Move the hair line along to 90 yards and read the result under it as 3.75 volts.

These scales can also be used to find the cross sectional area for a fixed voltage drop, the formula being :

$$\text{Area in sq. ins.} = \frac{\text{Length in yards} \times \text{Current in amps} \times \text{Resistance of copper}}{\text{Voltage Drop}}$$

In this case also, the slide automatically multiplies by the resistance of copper.

EXAMPLE : What is the area required to carry 120 amps 50 yards with a drop of 2.5 volts ?

Set the hair line to 2.5 volts and bring 50 yards on the length scale to correspond with it, then corresponding with 120 amps will be found the answer .0582 sq. inches.

EXAMPLE : What is the area required to carry 6.2 amps 600 yards for a voltage drop of 1 volt ?

As 6.2 amps cannot be found directly on the rule, the problem can be re-stated as the area to carry 62 amps 60 yards for 1 volt drop, because the drop varies directly as the current and the length of cable.

Set the hair line to 1 volt and bring 60 yards to correspond with it, and read off the area .090 sq. inches opposite 62 amps.

All of the above examples have been for the standard resistance of 60° F. Voltage drops for the other temperatures can readily be found by the use of the *Temperature* scale.

EXAMPLE : Find the area required to carry 6.2 amps 600 yards for a voltage drop of 1 volt at a temperature of 90° F.

As the resistance and voltage drop increase with temperature, the permissible voltage drop at 90° F. will be reduced for the standard temperature of 60° F.

Set the hair line to 1 volt and bring 90° on the *Temperature*

scale to correspond with it. Then move the hair line to  $60^\circ$  and bring 60 yards to correspond with it and read off .096 sq. inches opposite 62 amps.

#### EXERCISES ON ELECTRICAL PROBLEMS.

- Exercise 68. What is the output of a dynamo of 90% efficiency with an input of 25 horse-power ?
69. What is the efficiency of a dynamo giving 15 kilowatts with an input of 27 horse-power ?
70. What is the efficiency of a 25 horse-power motor having an input of 20 kilowatts ?
71. What is the input of a 30 horse-power motor which has an efficiency of 85%.
72. What is the voltage drop when 20 amps flow along a conductor 45 yards in length having a cross-sectional area of .0030 sq. ins. ?
73. What is the voltage drop when 25 amps flow along a conductor 1500 yards in length, having a cross-sectional area of .35 sq. ins. ?

#### ANSWERS TO EXERCISES

- |             |                 |                                                    |
|-------------|-----------------|----------------------------------------------------|
| 1. 9.8      | 26. 23.45       | 51. .01196                                         |
| 2. 12.6     | 27. 168.8       | 52. .114                                           |
| 3. 8500.0   | 28. 1.3         | 53. 2.94                                           |
| 4. 2600.0   | 29. .0552       | 54. 4.672                                          |
| 5. 15.8     | 30. 4.096       | 55. .610                                           |
| 6. 975.0    | 31. 19683.0     | 56. 2.364                                          |
| 7. 607.5    | 32. .000,002,74 | 57. 842.5                                          |
| 8. 59.47    | 33. 941.2       | 58. 4.55                                           |
| 9. 74.1     | 34. 5.94        | 59. 2.095                                          |
| 10. 1.95    | 35. 1.86        | 60. 58.0                                           |
| 11. 152.5   | 36. 3.5         | 61. 34.6                                           |
| 12. 67.5    | 37. .14         | 62. 36.1                                           |
| 13. 2.64    | 38. .00364      | 63. 227.01                                         |
| 14. .947    | 39. 13.89       | 64. 18.3                                           |
| 15. 2.04    | 40. .1037       | 65. 156.3                                          |
| 16. .717    | 41. .000494     | 66. $A = 56^\circ$<br>$B = 73^\circ$<br>$b = 83.0$ |
| 17. .745    | 42. 770.0       | 67. $v = 42.8$<br>$h = 183.0$                      |
| 18. 5.79    | 43. .181        | 68. 16.8 K.W.                                      |
| 19. .548    | 44. .1085       | 69. 74.5%                                          |
| 20. 411.0   | 45. .359        | 70. 93.3%                                          |
| 21. 602.0   | 46. 6.45        | 71. 26.3 K.W.                                      |
| 22. 25.5    | 47. .000,008,7  | 72. 7.3 Volts                                      |
| 23. 2209.0  | 48. 5128.2      | 73. 2.6 Volts                                      |
| 24. .000676 | 49. .00526      |                                                    |
| 25. 3.42    | 50. .1048       |                                                    |