

1. Move HL over 18 on  $R_1$ . HL is now over  $(18)^2$  on D.
2. Slide 29 on C under HL. Move HL over left index of C.
3. Under HL read "3342" on  $R_1$ . Answer is **3.342**.

*Verify the following:*

$$1. \frac{26}{\sqrt{6.2}} = 10.44$$

$$5. \frac{10}{\sqrt{15.4}} = 2.55$$

$$2. \frac{68}{\sqrt{14}} = 18.2$$

$$6. \frac{12.5}{(7.25)^{1/2}} = 4.64$$

$$3. 134 \div \sqrt{210} = 9.25$$

$$7. \frac{453}{(4150)^{1/2}} = 7.03$$

$$4. \frac{43}{\sqrt{55}} = 5.80$$

$$8. 2.44 \div (.0382)^{1/2} = 12.5$$

### Exercise 12-1

$$1. 5.2 \times \sqrt{4.8} =$$

$$11. \frac{120}{\sqrt{340}} =$$

$$2. 1.3 \times \sqrt{2.5} = \text{"206"}$$

$$12. \frac{7.4}{\sqrt{2.7}} = \text{"450"}$$

$$3. 23 \times \sqrt{19} =$$

$$13. 12.5 \times \sqrt{423} =$$

$$4. 14 \times \sqrt{31} = \text{"780"}$$

$$14. 8.22 \times \sqrt{0.725} = \text{"700"}$$

$$5. \frac{\sqrt{7.3}}{2.1} =$$

$$15. \frac{\sqrt{5650}}{10.2} =$$

$$6. \frac{\sqrt{5.7}}{1.6} = \text{"1492"}$$

$$16. \frac{\sqrt{2450}}{15.6} = \text{"317"}$$

$$7. \frac{\sqrt{71}}{4.8} =$$

$$17. \frac{230}{\sqrt{108}} =$$

$$8. \frac{\sqrt{43}}{3.2} = \text{"205"}$$

$$18. \frac{9.25}{\sqrt{17.2}} = \text{"223"}$$

$$9. \frac{14}{\sqrt{1.9}} =$$

$$19. 41.2 \times (.061)^{1/2} =$$

$$10. \frac{34}{\sqrt{21}} = \text{"742"}$$

$$20. .045 \times (735)^{1/2} = \text{"1220"}$$

21.  $\frac{(.037)^{1/2}}{2.11} =$

22.  $\frac{(.0071)^{1/2}}{1.85} = \text{"456"}$

23.  $\frac{1}{\sqrt{3.75}} =$

24.  $\frac{1}{\sqrt{.077}} = \text{"360"}$

25.  $21.6 \times \sqrt{51.6} =$

26.  $7.44 \times \sqrt{2.03} = \text{"1060"}$

27.  $\frac{\sqrt{344}}{6.48} =$

28.  $\frac{\sqrt{41.2}}{2.75} = \text{"233"}$

29.  $0.821 \times \sqrt{18.4} =$

30.  $\sqrt{78.2} \times 41.6 = \text{"368"}$

31.  $\frac{\sqrt{126}}{4.66} =$

32.  $\frac{\sqrt{5670}}{2.41} = \text{"312"}$

33.  $\frac{25.8}{(0.73)^{1/2}} =$

34.  $\frac{456}{\sqrt{234}} = \text{"298"}$

35.  $\frac{36.4}{\sqrt{9.55}} =$

36.  $\frac{2.22}{\sqrt{0.431}} = \text{"338"}$

37.  $(138)^{1/2} \times 2.07 =$

38.  $\pi \times \sqrt{0.811} = \text{"283"}$

39.  $\frac{\sqrt{10.45}}{0.823} =$

40.  $\frac{\sqrt{65,400}}{27.2} = \text{"940"}$

41.  $.0124 \times \sqrt{1545} =$

42.  $1.78 \times (473)^{1/2} = \text{"387"}$

43.  $\frac{\sqrt{51,200}}{43.6} =$

44.  $\frac{52.1}{(17.2)^{1/2}} = \text{"1256"}$

45.  $\frac{(.0941)^{1/2}}{0.423} =$

46.  $456 \times \sqrt{.000714} = \text{"1218"}$

47.  $\frac{\sqrt{.0555}}{1.755} =$

48.  $126 \times \sqrt{.0545} = \text{"294"}$

49.  $.0452 \times \sqrt{84,300} =$

50.  $\frac{\sqrt{165}}{.0144} = \text{"892"}$

51.  $\frac{\sqrt{495}}{36.7} =$

52.  $\frac{\sqrt{0.204}}{821} = \text{"550"}$

53.  $\frac{\sqrt{9250} \times 10^4}{3400} =$

54.  $\frac{\sqrt{175,000}}{6.55 \times 10^3} = \text{"639"}$

55.  $\frac{(75.6)^{1/2}}{0.334} =$

56.  $\frac{204}{\sqrt{74.2}} = \text{"237"}$

57.  $\frac{1030}{\sqrt{4570}} =$

58.  $(69.1)^{-1/2} = \text{"1203"}$

59.  $(.00504)^{-1/2} =$

## 12.2 Further operations with square roots

Example 1:  $\sqrt{2.7 \times 6.3} = ?$

*Procedure with A-B scales:*

Think of the given expression as  $\sqrt{2.7} \times \sqrt{6.3}$ .

1. Set left index of B opposite 27 on A-left. Note that left C index is now opposite  $\sqrt{2.7}$  on D.
2. Move HL over 63 on B-left. The hairline is now over  $\sqrt{6.3}$  on C; hence, we have performed the desired multiplication on the C-D scales.
3. Under HL read "412" on D. Answer is **4.12**.

*Procedure with R scale:*

The expression under the radical sign is evaluated on the C-D scales and the square root of this result is read on the appropriate R scale:

1. Set right index of C opposite 63 on D.
2. Move HL over 27 on C. The product under the radical sign is now under the HL on D; the square root of this is under HL on either  $R_1$  or  $R_2$ . By approximation, it is evident that answer is on  $R_2$ .
3. Under HL read "4125" on  $R_2$ . Answer is **4.125**.

Example 2:  $\sqrt{\frac{21.7 \times 0.165}{6.24}} = ?$

*Procedure with A-B scales:*

Think of expression as:  $\frac{\sqrt{21.7} \times \sqrt{0.165}}{\sqrt{6.24}}$

1. Move HL over 217 on A-right. HL is now over  $\sqrt{21.7}$  on D.
2. Slide 624 on B-left under HL. This positions  $\sqrt{6.24}$  on C under HL. Thus, we have divided  $\sqrt{21.7}$  by  $\sqrt{6.24}$  on the C-D scales. Now multiply by  $\sqrt{0.165}$ :
3. Move HL over 165 on B-right. HL is now over  $\sqrt{0.165}$  on C; hence, result is under HL on D.
4. Under HL read "758" on D.

Approximating for decimal point:

$$\sqrt{\frac{21.7 \times 0.165}{6.24}} \approx \sqrt{\frac{20 \times 0.2}{6}} = \sqrt{\frac{4}{6}} \approx \sqrt{0.7} \approx 0.8$$

Answer must be **0.758**.

(Again, you are reminded that the quantities under the radical sign must be set on the proper sections of the A and B scales.)

*Procedure with R scale:*

First approximate the answer to be about 0.8. Then evaluate the expression under the radical sign on the C-D scales, and read the square root on the appropriate R scale.

1. Set left index of C opposite 165 on D.
2. Move HL over 217 on C.
3. Slide 624 on C under HL. The expression under the radical sign has now been evaluated and the result is opposite C index on D. We wish to extract the square root of this:
4. Move HL over right index of C.
5. Under HL read "758" on  $R_2$ . Answer is **0.758**.

*Verify the following:*

$$1. \sqrt{6.45 \times 23.9} = 12.41$$

$$6. \sqrt{.077 \times 266} = 4.53$$

$$2. \sqrt{\frac{602}{46.5}} = 3.60$$

$$7. \frac{\sqrt{127 \times .063}}{\sqrt{20.5}} = 0.625$$

$$3. \sqrt{\frac{172}{4.95}} = 5.90$$

$$8. \sqrt{\frac{31.2 \times 4.60}{2.55 \times 0.44}} = 11.31$$

$$4. \sqrt{\frac{5.25 \times 31.2}{2.44}} = 8.20$$

$$9. \frac{\sqrt{23,500}}{\sqrt{0.912 \times 413}} = 7.90$$

$$5. \frac{\sqrt{55.5}}{\sqrt{.077}} = 26.8$$

$$10. \sqrt{\frac{4570}{23.1 \times 6.27}} = 5.62$$

Example 3:  $\frac{\sqrt{5210 \times 0.410}}{\sqrt{.0755}} = ?$

*Procedure with A-B scales:*

1. Move HL over 521 on A-right.
2. Slide 755 on B-left under HL. We have now divided  $\sqrt{5210}$  by  $\sqrt{.0755}$ . The operation has taken place on C-D, and the quotient is opposite left C index on D. To multiply by 0.410 on the C-D scales, we must now interchange indexes. However, this may be avoided by continuing on the folded scales:
3. Turn rule over and move HL over 410 on CF.
4. Under HL read "1077" on DF.

Approximating for the decimal point:

$$\frac{\sqrt{52 \ 10} \times 0.410}{\sqrt{.07 \ 55}} \approx \frac{70 \times 0.4}{0.3} = \frac{280}{3} \approx 90$$

Answer must be **107.7**.

*Procedure with R scale:*

You may first divide 5210 by .0755 on C-D and find the square root of the result on R. Then transfer this back to D and multiply by 0.410.

Alternatively, the expression may be evaluated as  $\sqrt{\frac{5210 \times (0.410)^2}{.0755}}$ .

1. Move HL over 410 on  $R_2$ . This puts  $(0.410)^2$  on D.
2. Slide 755 on C under HL.
3. Move HL over 521 on C. Answer is now under HL on  $R_1$  or  $R_2$ ; however, a rough approximation indicates that the answer must be on  $R_1$ .
4. Under HL read "1077" on  $R_1$ . Answer is **107.7**.

Example 4:  $\frac{286}{11.2 \times \sqrt{3.75}} = ?$

*Procedure with A-B scales:*

1. Move HL over 286 on D.
2. Slide 375 on B-left under HL. Now go to the folded scales:
3. Turn rule over and move HL over 112 on CIF.
4. Under HL read "1319" on DF. Answer is **13.19**.

*Procedure with R scale:*

You may first find  $\sqrt{3.75}$  on R, then evaluate as a combined operation on C-D.

Alternatively, the expression may be evaluated as  $\sqrt{\frac{(286)^2}{(11.2)^2 \times 3.75}}$ .

Verify that answer is **13.19**.

Example 5:  $\frac{\sqrt{28 \times 8.4}}{(2.2)^2} = ?$

When both square roots and squares are involved, the squares are simply treated as products.

Verify that  $\frac{\sqrt{28 \times 8.4}}{(2.2)^2} = \frac{\sqrt{28 \times 8.4}}{2.2 \times 2.2} = 3.17$

*Verify the following:*

1.  $\frac{\sqrt{5.4 \times 23}}{\sqrt{11}} = 16.11$

4.  $\frac{\sqrt{5200 \times (3.4)^2}}{\sqrt{.073}} = 3090$

2.  $\frac{\sqrt{32.4 \times 1.46}}{2.15} = 3.86$

5.  $\frac{12.4}{0.362 \times \sqrt{.0017}} = 830$

3.  $\frac{250}{\sqrt{2.6 \times 15.4}} = 39.5$

6.  $\frac{\sqrt{53.5 \times .026}}{375} = .00315$

Example 6:  $\frac{(8.5)^{3/2}}{2.6} = ?$

Verify that  $\frac{(8.5)^{3/2}}{2.6} = \frac{8.5 \times \sqrt{8.5}}{2.6} = 9.53$

Example 7:  $(2.3)^{5/2} = ?$

Verify that  $(2.3)^{5/2} = (2.3)^2 \times \sqrt{2.3} = \mathbf{8.03}$

*Verify the following:*

1.  $37.5 \times (11.5)^{3/2} = 1462$

2.  $2.7 \times (46)^{3/2} = 843$

3.  $\frac{(52.5)^{3/2}}{6.44} = 59.1$

4.  $\frac{450}{(6.45)^{3/2}} = 27.5$

5.  $(12.5)^{5/2} = 553$

6.  $(6.7)^{5/2} \div 3.7 = 31.4$

The foregoing techniques may be summarized as follows:

*Combined operations with square roots*

*Procedure with A-B scales:*

1. Numbers under radical signs are each set on the proper sections of the A-B scales, thus locating the square roots on C-D.
2. Numbers not under radical signs are set on C-D, and the actual operation takes place on the C-D scales.

*Procedure with R scale:*

1. Square roots may first be evaluated on R, then transferred back to C-D if they are to be further multiplied or divided.
2. Alternatively, if the given expression is all contained under a single radical sign (or if it is put into this form), then the expression is evaluated on C-D with the square root appearing on the appropriate R scale.

**Exercise 12-2**

1.  $\sqrt{\frac{1.8 \times 42}{2.9}} =$

2.  $\sqrt{\frac{56 \times 3.4}{23}} = \text{“288”}$

3.  $\sqrt{\frac{120 \times 4.5}{37}} =$

4.  $\sqrt{6.2 \times 3.4} = \text{“459”}$

5.  $\sqrt{15 \times 8.4} =$

6.  $\sqrt{\frac{73}{2.1 \times 3.5}} = \text{"315"}$
7.  $\frac{\sqrt{220}}{\sqrt{6.2} \times \sqrt{10.5}} =$
8.  $\frac{\sqrt{31}}{\sqrt{3.8}} = \text{"286"}$
9.  $\sqrt{\frac{260}{12.5}} =$
10.  $\left(\frac{14}{37}\right)^{1/2} = \text{"615"}$
11.  $\sqrt{2.7 \times 5.6 \times 3.1} =$
12.  $\sqrt{43 \times 0.35 \times 8.7} = \text{"1145"}$
13.  $\frac{14 \times \sqrt{23}}{\sqrt{3.5}} =$
14.  $\frac{2.7 \times \sqrt{8.2}}{\sqrt{19}} = \text{"1775"}$
15.  $6.2 \times \sqrt{\frac{210}{5.7}} =$
16.  $\frac{\sqrt{5.1} \times \sqrt{31}}{2.4} = \text{"524"}$
17.  $\frac{\sqrt{145} \times \sqrt{3.7}}{13} =$
18.  $\frac{(61.2)^{3/2}}{32.5} = \text{"1475"}$
19.  $0.73 \times (11.5)^{3/2} =$
20.  $1.82 \times (0.83)^{3/2} = \text{"1376"}$
21.  $\frac{12 \times 17}{\sqrt{5.1}} = \text{"903"}$
22.  $\frac{5.4 \times 75}{\sqrt{29}} =$
23.  $\frac{\sqrt{340} \times 6.2}{23} = \text{"497"}$
24.  $\frac{170 \times \sqrt{0.42}}{53} =$
25.  $\frac{(.00717)^{1/2} \times 0.811}{.00406} = \text{"1691"}$
26.  $\frac{7840}{\sqrt{21.2} \times 40.6} =$
27.  $\frac{1}{\sqrt{0.814} \times .0243} = \text{"456"}$
28.  $\frac{37.4}{\sqrt{.0524} \times 6.44} =$
29.  $\frac{2.76 \times 525}{\sqrt{40,800}} = \text{"717"}$
30.  $\frac{1.75 \times 4.77}{\sqrt{.0335}} =$
31.  $\frac{23.4 \times 10^4}{(7430)^{1/2}} = \text{"272"}$
32.  $\frac{23.0 \times (8.45)^{3/2}}{6.32} =$
33.  $\frac{246}{4.82 \times (8.25)^{3/2}} = \text{"215"}$
34.  $\sqrt{.00712} \times \sqrt{1.245} =$
35.  $\sqrt{.0831} \times 3450 = \text{"1694"}$
36.  $\frac{\sqrt{4.73}}{\sqrt{19.2}} =$
37.  $\frac{\sqrt{59.2}}{\sqrt{.00345}} = \text{"1310"}$
38.  $\frac{\sqrt{621}}{\sqrt{21.2} \times \sqrt{10.4}} =$
39.  $\frac{\sqrt{7.25} \times \sqrt{31.6}}{\sqrt{85.2}} = \text{"1640"}$
40.  $(.0431)^{-1/2} =$
41.  $\frac{\sqrt{902} \times \sqrt{0.412}}{\sqrt{.0521}} = \text{"845"}$

$$42. \sqrt{\frac{37.2}{4.75 \times .0714}} =$$

$$43. \sqrt{\frac{.00718 \times 5760}{.0436}} = \text{"308"}$$

$$44. \sqrt{\frac{.0643 \times 384}{0.524}} =$$

$$45. \sqrt{\frac{342}{12.7 \times .0615}} = \text{"209"}$$

$$46. \frac{2.46 \times \sqrt{41.6}}{\sqrt{714}} =$$

$$47. \frac{375 \times \sqrt{0.617}}{\sqrt{2070}} = \text{"647"}$$

$$48. 0.432 \times \sqrt{\frac{27,500}{.0643}} =$$

$$49. \frac{\sqrt{69.2} \times \sqrt{.00316}}{23.4} = \text{"200"}$$

$$50. \frac{42.1 \times \sqrt{3.71}}{\sqrt{0.743}} =$$

$$51. 6.28 \times \sqrt{\frac{143}{32.2}} = \text{"1323"}$$

$$52. 2.46 \times \sqrt{\frac{375}{19.2}} =$$

$$53. \frac{3.44 \times \sqrt{0.611}}{\sqrt{29.4}} = \text{"496"}$$

$$54. \sqrt{\frac{9.36 \times 10^9}{5850}} = \text{"1265"}$$

$$55. \frac{6.19 \times 10^5}{\sqrt{3.77 \times 10^7}} =$$

$$56. \frac{\sqrt{.075 \times 463}}{7.42} = \text{"794"}$$

$$57. \frac{422}{\sqrt{7.06} \times \sqrt{213}} =$$

$$58. (2.4)^2 \times \sqrt{30.4} = \text{"318"}$$

$$59. (3.7)^2 \times \sqrt{0.86} =$$

$$60. \frac{(11.5)^2}{\sqrt{235}} = \text{"863"}$$

$$61. \frac{(0.76)^2}{\sqrt{14.25}} =$$

$$62. (31.2)^{5/2} = \text{"5440"}$$

$$63. (.0072)^{5/2} =$$

$$64. \frac{2.3 \times 360}{(6.2)^{5/2}} = \text{"865"}$$

$$65. \frac{(0.645)^{5/2}}{1.82} =$$

$$66. \frac{(1.63)^{5/2}}{26.5} = \text{"1280"}$$

$$67. \frac{\sqrt{3450}}{(6.35)^2} = \text{"1458"}$$

$$68. (5.2)^2 \times \sqrt{\frac{51.2}{6.31}} =$$

$$69. \frac{(.00206 \times 39.6)^{1/2}}{0.564} = \text{"507"}$$

$$70. \frac{\sqrt{2160} \times \sqrt{7.25}}{4.89 \times 0.564} =$$

$$71. \frac{43.7 \times \sqrt{.0823}}{\sqrt{71.2} \times 0.821} = \text{"1810"}$$

$$72. \frac{27.6 \times \sqrt{4750}}{\sqrt{157} \times 4.67} =$$

$$73. \left( \frac{37.2 \times .0168}{4.68 \times .00913} \right)^{1/2} = \text{"383"}$$

$$74. \left( \frac{125 \times 5720}{.0823 \times 7.11} \right)^{1/2} =$$

75. Evaluate  $y = 3.6\sqrt{x}$  for  $x = 1.5, 3.6, 21.7,$  and  $60.5$ .

76. Evaluate  $y = 0.74\sqrt{x}$  for  $x = 15, 75, 250,$  and  $1450$ .

77. Evaluate  $y = \frac{\sqrt{x}}{2.6}$  for  $x = 3, 7, 11,$  and  $45.$

78. Evaluate  $y = \frac{64}{\sqrt{x}}$  for  $x = 0.5, 8.5, 46.5,$  and  $175.$

79. Evaluate  $y = (26.4)x^{-1/2}$  for  $x = 0.70, 1.85, 7.40,$  and  $13.6.$

80. Evaluate  $y = \frac{150}{\pi \sqrt{x}}$  for  $x = 5.75, 12.6, 57.5,$  and  $210.$

### 12.3 Operations with cube roots and fourth roots

Example 1:  $\sqrt[3]{12} \times 22 = ?$

1. Move HL over 12 on K-middle. This puts HL over  $\sqrt[3]{12}$  on D. We may now multiply by 22 on C-D scales:
2. Slide 22 on CI under HL.
3. Opposite right index of C read "504" on D. Answer is **50.4**.

Example 2:  $\frac{\sqrt[3]{410}}{6.20} = ?$

1. Move HL over 410 on K-right.
2. Slide 62 on C under HL.
3. Opposite left index of C read "1198" on D. Answer is **1.198**.

Example 3:  $\frac{52}{\sqrt[3]{7.4}} = ?$

In this case, we divide  $\sqrt[3]{7.4}$  by 52 and read the reciprocal.

1. Move HL over 74 on K-left.
2. Slide 52 on C under HL.
3. Move HL over right index of C. The quotient is now under HL on D, and its reciprocal is under HL on DI.
4. Under HL read "267" on DI. Answer is **26.7**.  
(Note that reciprocal may also be read opposite left D index on C.)

*Verify the following:*

1.  $\sqrt[3]{4.5} \times 47 = 77.6$

2.  $\frac{\sqrt[3]{5450}}{2.64} = 6.66$

$$3. \frac{136}{\sqrt[3]{13.5}} = 57.2$$

$$5. \frac{2.76 \times 67.5}{\sqrt[3]{0.37}} = 260$$

$$4. \frac{\sqrt[3]{31.5} \times 16.2}{3.44} = 14.87$$

$$6. \frac{5210}{325 \times \sqrt[3]{.0460}} = 44.8$$

**Example 4:**  $1.7 \times \sqrt[3]{31.2} = ?$

Evaluate this as  $1.7 \times \sqrt{\sqrt{31.2}}$ .

*Procedure with A-B scales:*

1. Move HL over 312 on A-right.
2. Under HL read "559" on D; hence,  $\sqrt{31.2} = 5.59$ . Now take square root again.
3. Move HL over 559 on A-left. HL is now over  $\sqrt[3]{31.2}$  on D, and we can multiply by 1.7.
4. Slide 17 on CI under HL.
5. Opposite right index of C read "401" on D. Answer is **4.01**.

*Procedure with R scale:*

1. Use R scale twice to find  $\sqrt[4]{31.2} = \sqrt{\sqrt{31.2}} = 2.36$ .
2. Slide left index of C over 236 on D, and multiply by 1.7. Verify that answer is **4.01**.

On rules that have both the A scale and the double-length scale (R, Sq, or  $\sqrt{\quad}$ ), you may read  $\sqrt[4]{31.2}$  directly from A to R, transfer to D and multiply by 1.7.

**Example 5:**  $(175)^{5/4} = ?$

Verify that  $(175)^{5/4} = 175 \times (175)^{1/4} = \mathbf{636}$ .

**Example 6:**  $(43)^{3/4} = ?$

Verify that  $(43)^{3/4} = (43)^{1/4} \times (43)^{1/2} = \mathbf{16.8}$ .

*Verify the following:*

$$1. 4.6 \times \sqrt[3]{7.2} = 7.54$$

$$4. (48)^{5/4} = 126.5$$

$$2. 26.3 \times \sqrt[3]{2850} = 192.5$$

$$5. (0.82)^{5/4} = 0.78$$

$$3. \sqrt[3]{.056} \div 1.8 = 0.270$$

$$6. (640)^{3/4} = 127.2$$

### Exercise 12-3

$$1. \sqrt[3]{32} \times 4.6 =$$

$$2. \sqrt[3]{175} \times 8.5 = \text{"476"}$$

3.  $\frac{\sqrt[3]{45}}{2.7} =$
4.  $\frac{\sqrt[3]{560}}{8.2} = \text{"1005"}$
5.  $\frac{165}{\sqrt[3]{61}} =$
6.  $\frac{34.6}{\sqrt[3]{2.75}} = \text{"247"}$
7.  $\frac{16.3 \times \sqrt[3]{760}}{57.4} =$
8.  $\frac{\sqrt[3]{13.5 \times 2.46}}{1.075} = \text{"545"}$
9.  $\frac{\sqrt[3]{96}}{4.75 \times 0.74} =$
10.  $\frac{\sqrt[3]{1750}}{6.35 \times 2.48} = \text{"765"}$
11.  $\frac{1}{\sqrt[3]{4.6 \times .065}} =$
12.  $\frac{1}{\sqrt[3]{.065 \times 1.22}} = \text{"204"}$
13.  $2.2 \times \sqrt[3]{185} =$
14.  $10.6 \times \sqrt[3]{3650} = \text{"824"}$
15.  $0.72 \times \sqrt[3]{52} =$
16.  $345 \times \sqrt[3]{.00665} = \text{"985"}$
17.  $\frac{\sqrt[3]{.0465}}{0.622} =$
18.  $\frac{\sqrt[3]{74,000} \times 12.8}{5.22} = \text{"404"}$
19.  $(22.6)^{5/4} =$
20.  $(0.63)^{5/4} = \text{"561"}$
21.  $(3750)^{5/4} =$
22.  $\frac{(5.22)^{5/4}}{2.33} = \text{"339"}$
23.  $(9.2 \times 10^{-9})^{5/4} =$
24.  $(230)^{3/4} = \text{"591"}$
25.  $(17.5)^{3/4} =$
26.  $(0.445)^{3/4} = \text{"545"}$
27.  $(5.6 \times 10^{10})^{3/4} =$
28.  $\frac{(1.9 \times 10^{-9})^{3/4}}{2.3} = \text{"1252"}$
29.  $\sqrt[3]{4.6 \times 3.5} =$
30.  $\sqrt[3]{61.5 \times 15.3} = \text{"980"}$
31.  $\sqrt[3]{\frac{210}{4.8}} =$
32.  $\sqrt[3]{\frac{76}{105}} = \text{"898"}$
33.  $\frac{16.7 \times 2.83}{\sqrt[3]{77}} =$
34.  $\frac{0.425 \times 378}{\sqrt[3]{460}} = \text{"208"}$
35.  $(165)^{2/3} \times 3.77 =$
36.  $(740)^{2/3} \div 11.6 = \text{"705"}$
37.  $\frac{(27.5)^{2/3} \times 6.35}{2.55} =$
38.  $\sqrt[3]{145} \times \sqrt{6.3} = \text{"1320"}$
39.  $\frac{\sqrt[3]{1450}}{\sqrt{260}} =$
40.  $\frac{\sqrt{510} \times 3.72}{\sqrt[3]{3160}} = \text{"572"}$

## 12.4 Formula types

Example 1: 
$$\frac{\sqrt{1 + (1.46)^2}}{.0178 + (4.77)(0.133)} = ?$$

1. Verify that  $(1.46)^2 = 2.13$ .
2. Verify that  $4.77 \times 0.133 = 0.634$ .
3. Expression now becomes:

$$\frac{\sqrt{1 + 2.13}}{.0178 + 0.634} = \frac{\sqrt{3.13}}{0.6518} \approx \frac{\sqrt{3.13}}{0.652}$$

4. Verify that answer is **2.72**.

Example 2: 
$$\frac{650 \times [(14.6)^2 - (8.25)^2]^{3/2}}{4320} = ?$$

1. Verify that  $(14.6)^2 - (8.25)^2 = 213 - 68.0 = 145$ .
2. Expression now becomes:

$$\frac{650 \times (145)^{3/2}}{4320} = \frac{650 \times 145 \times \sqrt{145}}{4320}$$

3. Verify that answer is **263**.

## Exercise 12-4

1. 
$$\frac{\sqrt{17.8} - \sqrt{6.22}}{0.27 \times 3.32} =$$

2. 
$$\frac{\sqrt{1 + (2.7)^2}}{3.5 \times 1.7} =$$

3. 
$$\sqrt{\frac{17 \times 34}{2.6(1 + 34/19)}} =$$

4. 
$$\frac{360 \times [(21.3)^2 - (12.4)^2]^{3/2}}{1300} =$$

5. 
$$\frac{0.85}{\sqrt{1 - \left(\frac{1.7}{2.1}\right)^4}} =$$

6. 
$$32.2 \times \sqrt[4]{2.3 + (5.7)^2} =$$

7. 
$$\sqrt{\frac{32.2 \times [104 + (5.24)^2]}{26.7}} =$$

$$8. 21 \times 6.2 \times \left[ \frac{21 \times 6.2}{21 + 12.4} \right]^{2/3} =$$

$$9. \frac{(28.2)^2 - (11.7)^2}{6.35 + \pi(3.18)^{4/3}} =$$

$$10. \frac{\sqrt[3]{\frac{1}{.0032} + \frac{1}{.0075}}}{5.24 \times 10^7} =$$

In the following formulas, substitute the given data and evaluate:

$$11. d = \sqrt{pq(1/N_1 + 1/N_2)}$$

a.  $N_1 = 220, N_2 = 170, p = 0.65, q = 0.34$   
 b.  $N_1 = 435, N_2 = 260, p = 0.83, q = 0.57$

$$12. r = \sqrt{\frac{p_1 r_1^2 + p_2 r_2^2}{p_1 + p_2 - 2}}$$

a.  $p_1 = 23, p_2 = 17, r_1 = 0.36, r_2 = 0.17$   
 b.  $p_1 = 74, p_2 = 55, r_1 = 0.67, r_2 = 0.29$

$$13. R = \frac{\sqrt{s(s-a)(s-b)(s-c)}}{s}$$

a.  $a = 11.5, b = 7.6, c = 8.3, s = 13.7$   
 b.  $a = 21.6, b = 13.2, c = 12.4, s = 23.6$

$$14. t = \frac{r\sqrt{N-2}}{\sqrt{1-r^2}}$$

a.  $r = 0.37, N = 20$   
 b.  $r = 0.58, N = 16$

$$15. C = 2\pi\sqrt{\frac{a^2 + b^2}{2}}$$

a.  $a = 26.4, b = 18.7$   
 b.  $a = 475, b = 385$

$$16. S = \pi r \sqrt{r^2 + h^2}$$

a.  $r = 16.7, h = 21.3$   
 b.  $r = 5.84, h = 12.7$

$$17. Q = c\pi r^2 \sqrt{2gh}$$

a.  $c = 0.594, r = .081, g = 32.2, h = 15$   
 b.  $c = 0.655, r = 0.135, g = 32.2, h = 37$

$$18. N = 50\pi d^2 \sqrt{\frac{ds}{4}}$$

a.  $d = 4.12, s = .00078$   
 b.  $d = 12.75, s = .00342$

$$19. Q = \frac{1.486}{n} R^{2/3} S^{1/2}$$

$$\text{a. } n = .015, R = 2.24, S = .00037$$

$$\text{b. } n = .034, R = 13.1, S = .0053$$

$$20. S = \pi (r_1 + r_2) \sqrt{h^2 + (r_1 - r_2)^2}$$

$$\text{a. } r_1 = 13.6, r_2 = 7.75, h = 8.18$$

$$\text{b. } r_1 = 63.2, r_2 = 28.6, h = 25.2$$

$$21. d = \frac{Pb \sqrt{3}}{27 EIL} (L^2 - b^2)^{3/2}$$

$$\text{a. } I = 11.4, P = 4200, b = 24.5, L = 72, E = 30 \times 10^6$$

$$\text{b. } I = 7.35, P = 2650, b = 14.7, L = 48, E = 30 \times 10^6$$

$$22. T = \frac{2A_t}{CA_0 \sqrt{2g}} (h_1^{1/2} - h_2^{1/2})$$

$$\text{a. } A_t = 13.3, A_0 = 0.74, C = 0.85, g = 32.2, h_1 = 7, h_2 = 5$$

$$\text{b. } A_t = 21.2, A_0 = 0.66, C = 0.79, g = 32.2, h_1 = 11, h_2 = 7$$

$$23. W = \sqrt{\frac{3EI}{L^3 (M + 0.24m)}}$$

$$\text{a. } E = 30 \times 10^6, I = 13.2, L = 86, M = 10.7, m = 4.6$$

$$\text{b. } E = 30 \times 10^6, I = 22.7, L = 64, M = 14.6, m = 8.55$$

$$24. f = 0.4 \sqrt{\frac{EIg}{WL^3}}$$

$$\text{a. } E = 10^7, I = 2.34, g = 386, L = 76.5, W = 1000$$

$$\text{b. } E = 30 \times 10^6, I = 11.5, g = 386, L = 58.0, W = 12,500$$

$$25. r = \frac{CT_m - ph}{\sqrt{CT_1 - p^2} \sqrt{CT_2 - h^2}}$$

$$\text{a. } C = 0.594, T_1 = 450, T_2 = 480, T_m = 465, p = 12, h = 15$$

$$\text{b. } C = 0.655, T_1 = 510, T_2 = 550, T_m = 530, p = 14, h = 17$$

## Chapter 13

### REVIEW EXERCISES

The following exercise sets are designed to give you additional drill with the techniques and scales covered in Chapters 1 through 12.

#### Exercise 13-1

1.  $\frac{472}{17.7 \times 2150} =$

2.  $\frac{x}{3.62} = \frac{365}{742}; x =$

3.  $\sqrt{.0615} =$

4.  $\frac{\sqrt[3]{260}}{\pi} =$

5.  $365 \times (.071)^{1/2} =$

6.  $\frac{\sqrt{128}}{5.24} =$

#### Exercise 13-2

1.  $11.7 \times (5.24)^2 =$

2.  $\sqrt[3]{.0000416} =$

3.  $(275)^{-1/2} =$

4.  $\frac{.00428 \times 1160}{6.04 \times .0848} =$

5.  $\sqrt{\frac{31.6}{4.66 \times .0685}} =$

6. Given three circles with diameters 2.44, 15.2, and 46.5 respectively.  
 a. Find circumference of each; b. find area of each.

**Exercise 13-3**

1.  $\frac{1}{.00266} =$

2.  $\frac{22.7}{\sqrt{0.425}} =$

3.  $\sqrt[4]{18.2} =$

4.  $\sqrt{2.13 \times 10^{11}} =$

5.  $\frac{.0744}{\frac{1}{350} + \frac{1}{240} + \frac{1}{560}} =$

6.  $1.8 \times 2.3 \times 2.2 \times 0.78 =$

**Exercise 13-4**

1.  $\frac{1230}{\pi \times 6.82} =$

2.  $\frac{1640}{V} = \frac{4.15}{.0633}; V =$

3.  $\sqrt[3]{39,000} =$

4.  $(315)^{2/3} =$

5.  $\frac{\sqrt{.00412 \times 722}}{3.78} =$

6.  $\frac{58.3}{\sqrt[3]{13.6}} =$

**Exercise 13-5**

$$1. \frac{5.86}{2.77} = \frac{R_1}{.00505} = \frac{R_2}{0.227}; R_1 = \quad, R_2 =$$

$$2. \frac{\pi}{(.064)^2} \left[ \frac{1.75}{72.3} - \frac{1.48}{84.6} \right] =$$

$$3. \text{ Given: } S = \frac{P}{A} + \frac{Mc}{I}$$

Find  $S$  when  $P = 6500$ ,  $A = 4.77$ ,  $M = 4650$ ,  $c = 2.60$ , and  $I = 8.75$ .

$$4. \frac{915 \times 84.7}{.0117 \times 128 \times 246} =$$

$$5. \sqrt[3]{.00575} \times 4.66 =$$

$$6. \text{ Given: } y = 3.6 \sqrt{x}. \text{ Find } y \text{ when } x = 1.8, 6.7, 41, \text{ and } 116.$$

**Exercise 13-6**

$$1. \text{ Given the equation, } y = 16.3x. \text{ Find } y \text{ when } x \text{ takes the values } 1.45, 3.85, 7.45, \text{ and } 9.25.$$

$$2. \frac{16.8 \times 1300 \times 10^{-14}}{27.5 \times 33.6 \times 6.03} =$$

$$3. \frac{(3.70 \times 41.2)^2}{57.2} =$$

$$4. \sqrt{31.4 \times 10^{-9}} =$$

$$5. \text{ The stretch of a spring is proportional to the applied force. If a force of 25 lbs stretches the spring 3.8 inches, find:}$$

a. stretch corresponding to 5.8 lbs, 18 lbs, and 46 lbs.

b. force corresponding to 0.75 inches, 2.1 inches, and 5.4 inches.

$$6. \sqrt{\frac{16 \times 450}{4.75(1 - 26/73)}} =$$

**Exercise 13-7**

$$1. \frac{\sqrt{4150 \times .0627}}{\sqrt{.0633}} =$$

$$2. \frac{.00423}{F} = \frac{0.684}{1315}; F =$$

3.  $\sqrt[3]{2.46 \times 10^9} =$

4.  $\frac{10^{13}}{.0754 \times 132 \times 2.65} =$

5.  $56(0.82)^5(0.18)^3 =$

6.  $\frac{.0815 \times 1065 \times 5.30 \times .0645}{436 \times .00358} =$

**Exercise 13-8**

1.  $\frac{(\sqrt{52.3} + \sqrt{21.7})^2}{3.14} =$

2.  $\frac{(46.2)^{3/2}}{2.44} =$

3.  $\frac{(.00564)^2 \times 322 \times 10^{23}}{(.0714)^2} =$

4.  $\frac{.0722 - \sqrt{.00357}}{3.42} =$

5. Given:  $C = 2\pi \sqrt{\frac{a^2 + b^2}{2}}$

Find  $C$  when  $a = 13.7$  and  $b = 8.75$ .

6.  $(2.17)^{5/2} =$

## Chapter 14

# THE TRIGONOMETRIC FUNCTIONS (S, ST, AND T SCALES)

### 14.1 Some important relations

In this chapter we shall make use of the following trigonometric relations:

*The reciprocal relations:*

$$\cos x = 1/\sin x$$

$$\sec x = 1/\cos x$$

$$\cot x = 1/\tan x$$

*The complementary relations:*

$$\cos x = \sin(90^\circ - x)$$

$$\cot x = \tan(90^\circ - x)$$

$$\csc x = \sec(90^\circ - x)$$

### 14.2 The sine (S and ST scales)

The S and ST scales are normally located on the slide, and the numbers on these scales represent angles in degrees (K & E uses the designation "SRT" instead of "ST"). You will observe that the ST scale ranges from about  $0.573^\circ$  to  $5.74^\circ$ , and the S scale continues from  $5.74^\circ$  to  $90^\circ$ ; thus, the two scales together form one continuous scale. You should carefully study the divisions on these scales so that you can quickly locate any angle with the hairline. On some slide rules, the subdivisions are such that fractions of a degree may be estimated in minutes; however, the trigonometric scales on most modern rules are subdivided in decimal fractions of a degree.

For angles in the range of the S scale ( $5.74^\circ$  to  $90^\circ$ ), the sine lies between 0.1 and 1.0. For angles in the ST range ( $0.573^\circ$  to  $5.74^\circ$ ), the sine is between .01 and 0.1. The S and ST scales are related to the C scale in the following way:

*To find  $\sin x$ :*

1. Move HL over  $x$  on **S** or **ST**.
2. Under HL read  $\sin x$  on **C** (or **D** if rule is closed).

*To place the decimal point:*

1. If  $x$  is located on **S**,  $\sin x$  is between 0.1 and 1.0.
2. If  $x$  is located on **ST**,  $\sin x$  is between .01 and 0.1.

When using the rule to simply read off trigonometric functions, it is a good idea to close the rule with the C and D indexes aligned. In this way, readings can be made on either C or D, and you eliminate the possibility of reading the wrong scale.

**Example 1:**       $\sin 26^\circ = ?$

1. Close rule and move HL over  $26^\circ$  on S.
2. Under HL read "438" on C or D.

Angle is located on S; hence, result lies between 0.1 and 1.0. Answer is **0.438**.

**Example 2:**       $\sin 3.25^\circ = ?$

1. Close rule and move HL over  $3.25^\circ$  on ST.
2. Under HL read "567" on C or D.

Angle is located on ST; hence, result lies between .01 and 0.1. Answer is **.0567**.

**Example 3:**       $\sin 36^\circ 37' = ?$

1. For rules with decimal subdivisions, we first divide  $37'$  by 60 to convert to a decimal fraction of a degree.
2. Verify that  $\sin 36^\circ 37' \approx \sin 36.6^\circ = \mathbf{0.596}$ .

**Verify the following:**

- |                              |                                |                                |
|------------------------------|--------------------------------|--------------------------------|
| 1. $\sin 36^\circ = 0.588$   | 4. $\sin 4.1^\circ = .0715$    | 7. $\sin 8^\circ 15' = 0.1435$ |
| 2. $\sin 79^\circ = 0.982$   | 5. $\sin 1.075^\circ = .01877$ | 8. $\sin 50^\circ 12' = 0.768$ |
| 3. $\sin 14.2^\circ = 0.245$ | 6. $\sin 0.73^\circ = .01274$  | 9. $\sin 2^\circ 40' = .0465$  |

It is now clear that we can read the sine directly for all angles between  $0.573^\circ$  and  $90^\circ$ . Functions of angles smaller than  $0.573^\circ$  are discussed in Appendix A.

### Exercise 14-1

- |                           |                           |
|---------------------------|---------------------------|
| 1. $\sin 39^\circ =$      | 16. $\sin 1.86^\circ =$   |
| 2. $\sin 14^\circ =$      | 17. $\sin 43' =$          |
| 3. $\sin 68.5^\circ =$    | 18. $\sin 35^\circ 12' =$ |
| 4. $\sin 10.2^\circ =$    | 19. $\sin 76^\circ =$     |
| 5. $\sin 45.6^\circ =$    | 20. $\sin 4.45^\circ =$   |
| 6. $\sin 2.3^\circ =$     | 21. $\sin 5^\circ 22' =$  |
| 7. $\sin 1.14^\circ =$    | 22. $\sin 14.35^\circ =$  |
| 8. $\sin 17.6^\circ =$    | 23. $\sin 6.05^\circ =$   |
| 9. $\sin 57.3^\circ =$    | 24. $\sin 62.4^\circ =$   |
| 10. $\sin 82^\circ =$     | 25. $\sin 12^\circ 17' =$ |
| 11. $\sin 30.5^\circ =$   | 26. $\sin 1^\circ 11' =$  |
| 12. $\sin 0.745^\circ =$  | 27. $\sin 85^\circ =$     |
| 13. $\sin 7.66^\circ =$   | 28. $\sin 20.6^\circ =$   |
| 14. $\sin 26.3^\circ =$   | 29. $\sin 0.585^\circ =$  |
| 15. $\sin 15^\circ 20' =$ | 30. $\sin 41^\circ 40' =$ |

### 14.3 The complementary scale

You will recall that two angles are complementary if their sum is  $90^\circ$ . Thus, the complement of  $24^\circ$  is  $66^\circ$ , the complement of  $31^\circ$  is  $59^\circ$ , and so on. Now, on most slide rules, both the angle and its complement are indicated at the major division marks on the S scale (on some rules the complementary angles are indicated in red). At the  $60^\circ$  mark, for example, you will also see  $30^\circ$  indicated; at the  $20^\circ$  mark you will see  $70^\circ$ . In using these complementary markings, we will refer to the "complementary scale." Note, especially, that the complementary scale increases from *right to left*. The presence of the complementary scale enables you to locate the complement of an angle directly on S without actually subtracting from  $90^\circ$ .

Most slide rules do not have the complementary markings on the ST scale; to locate the complement on this scale you must first subtract from  $90^\circ$ .

#### 14.4 The cosine

Here, we may use the complementary relation:  $\cos x = \sin(90^\circ - x)$ . Thus,  $\cos 25^\circ = \sin 65^\circ$ ,  $\cos 4^\circ = \sin 86^\circ$ , and so forth. Clearly, then, to find  $\cos x$  we need only to locate the complement of  $x$  on S or ST; we may then read the cosine directly on C. Note also that the complement of  $x$  can be located on S simply by moving the hairline over  $x$  on the complementary scale.

**Example 1:**  $\cos 62^\circ = ?$

The complementary relation is:  $\cos 62^\circ = \sin(90^\circ - 62^\circ) = \sin 28^\circ$ .

1. Close rule and move HL over  $62^\circ$  on the complementary scale of S. Note that HL is now automatically over the complementary angle ( $28^\circ$ ) on the direct S scale, thus saving you the trouble of subtracting from  $90^\circ$ .
2. Under HL read "469" on C or D. Complement is on S; hence, result is between 0.1 and 1.0. Answer is **0.469**.

This example illustrates the procedure:

*To find  $\cos x$ :*

1. Move HL over the *complement of  $x$*  on **S** or **ST**.
2. Under HL read  $\cos x$  on **C** (or **D** if rule is closed).

*To place the decimal point:*

1. If complement of  $x$  is on S,  $\cos x$  is between 0.1 and 1.0.
2. If complement of  $x$  is on ST,  $\cos x$  is between .01 and 0.1.

**Example 2:**  $\cos 46.3^\circ = ?$

1. Close rule and move HL over  $46.3^\circ$  on the complementary scale of S.
2. Under HL read "691" on C or D.

Complement is on S; hence, result is between 0.1 and 1.0. Answer is **0.691**.

**Example 3:**  $\cos 88.5^\circ = ?$

If there is no complementary scale on ST, we must first obtain the complement:  $90^\circ - 88.5^\circ = 1.5^\circ$ .

1. Close rule and move HL over  $1.5^\circ$  on ST.
2. Under HL read "262" on C or D.

Complement is on ST; hence, result is between .01 and 0.1. Answer is **.0262**.

*Verify the following:*

1.  $\cos 28^\circ = 0.883$

4.  $\cos 87.25^\circ = .0480$

2.  $\cos 64.5^\circ = 0.430$

5.  $\cos 46^\circ 15' = 0.692$

3.  $\cos 73^\circ 30' = 0.284$

6.  $\cos 85.2^\circ = .0837$

### Exercise 14-2

1.  $\cos 43^\circ =$

16.  $\cos 47^\circ 30' =$

2.  $\cos 67^\circ =$

17.  $\cos 87^\circ 30' =$

3.  $\cos 22^\circ =$

18.  $\cos 49^\circ 20' =$

4.  $\cos 75.2^\circ =$

19.  $\cos 73.6^\circ =$

5.  $\cos 58.4^\circ =$

20.  $\cos 31.5^\circ =$

6.  $\cos 37.7^\circ =$

21.  $\cos 55^\circ 36' =$

7.  $\cos 83.15^\circ =$

22.  $\cos 89^\circ 13' =$

8.  $\cos 16^\circ =$

23.  $\cos 28.4^\circ =$

9.  $\cos 89^\circ =$

24.  $\cos 42.8^\circ =$   
 $\sin 42.8^\circ =$

10.  $\cos 87.4^\circ =$

25.  $\cos 72.7^\circ =$   
 $\sin 72.7^\circ =$

11.  $\cos 89.24^\circ =$

26.  $\cos 18.6^\circ =$   
 $\sin 18.6^\circ =$

12.  $\cos 69.7^\circ =$

27.  $\cos 52^\circ 20' =$   
 $\sin 52^\circ 20' =$

13.  $\cos 81.45^\circ =$

28.  $\cos 34.3^\circ =$   
 $\sin 34.3^\circ =$

14.  $\cos 29.6^\circ =$

15.  $\cos 5^\circ =$

### 14.5 The tangent of angles less than $45^\circ$ (ST and T scales)

For the range covered by the ST scale, the angles are small enough so that the sine and tangent are substantially equal. At least, they are close enough so that we cannot dis-

tinguish the difference on the slide rule. Therefore, if the hairline is moved over an angle on ST, the reading on C corresponds to either the sine or the tangent. This, of course, accounts for the scale designation "ST" (sine or tangent).

**Example 1:**  $\tan 2.4^\circ = ?$

1. Close rule and move HL over  $2.4^\circ$  on ST.
2. Under HL read "419" on C or D. Answer is **.0419**.

For angles beyond the ST range, we must refer to the T scale. If you examine this scale, you will see that it extends from  $5.7^\circ$  to  $45^\circ$  (tangents of angles in this range will lie between 0.1 and 1.0). Using either the ST or T scales, we may read tangents directly on C:

*To find  $\tan x$  ( $x$  less than  $45^\circ$ ):*

1. Move HL over  $x$  on **T** or **ST**.
2. Under HL read  $\tan x$  on **C** (or **D** if rule is closed).

*To place the decimal point:*

1. If  $x$  is located on T,  $\tan x$  is between 0.1 and 1.0.
2. If  $x$  is located on ST,  $\tan x$  is between .01 and 0.1.

**Example 2:**  $\tan 21^\circ = ?$

1. Close rule and move HL over  $21^\circ$  on T.
2. Under HL read "384" on C or D.

Angle is located on T; hence, result is between 0.1 and 1.0. Answer is **0.384**.

**Verify the following:**

- |                               |                                 |
|-------------------------------|---------------------------------|
| 1. $\tan 17^\circ = 0.306$    | 7. $\tan 41^\circ 20' = 0.880$  |
| 2. $\tan 8.75^\circ = 0.1539$ | 8. $\tan 4^\circ 30' = .0785$   |
| 3. $\tan 28.6^\circ = 0.545$  | 9. $\tan 12^\circ 30' = 0.222$  |
| 4. $\tan 21.2^\circ = 0.388$  | 10. $\tan 32.2^\circ = 0.630$   |
| 5. $\tan 6.45^\circ = 0.1130$ | 11. $\tan 0^\circ 45' = .01309$ |
| 6. $\tan 3.24^\circ = .0565$  | 12. $\tan 1.65^\circ = .0288$   |

14.6 The tangent of angles greater than  $45^\circ$ 

Again referring to the T scale, you will observe that, just as on the S scale, the complementary angles are indicated. Thus, although the regular T scale extends only to  $45^\circ$ , the complementary scale on T ranges from  $45^\circ$  to about  $84.3^\circ$ .

Recall now that  $\cot x = \tan(90^\circ - x)$ ; hence, if the hairline is set over an angle on the complementary scale of T, we will read the cotangent directly on C. But the tangent is just the reciprocal of the cotangent; therefore, the tangent will be under the hairline on CI (or DI if rule is closed).

**Example 1:**  $\tan 63^\circ = ?$

1. Close rule and move HL over  $63^\circ$  on the complementary scale of T. The hairline is now over  $\cot 63^\circ$  on C, and over  $\tan 63^\circ$  on CI (or DI).
2. Under HL read "1963" on CI or DI.

The reading on C (or D) is between 0.1 and 1.0; hence, the reciprocal must be between 1 and 10. Answer is **1.963**.

The general procedure may be stated:

*To find  $\tan x$  ( $x$  greater than  $45^\circ$ ):*

1. Move HL over *complement of  $x$*  on **T** or **ST**.
2. Under HL read  $\tan x$  on **CI** (or **DI** if rule is closed).

*To place the decimal point:*

1. If complement of  $x$  is on T,  $\tan x$  is between 1 and 10.
2. If complement of  $x$  is on ST,  $\tan x$  is between 10 and 100.

**Example 2:**  $\tan 78.5^\circ = ?$

1. Close rule and move HL over  $78.5^\circ$  on the complementary scale of T.
2. Under HL read "492" on CI or DI.

Complement is on T; hence, result is between 1 and 10. Answer is **4.92**.

**Example 3:**  $\tan 88.8^\circ = ?$

First obtain the complement:  $90^\circ - 88.8^\circ = 1.2^\circ$ .

1. Close rule and move HL over  $1.2^\circ$  on ST.
2. Under HL read "477" on CI or DI.

Complement is on ST; hence, result is between 10 and 100. Answer is **47.7**.

Notice that by reading from the direct angle to C, or by reading from the complementary angle to CI, we are able to evaluate tangents of angles between  $0.573^\circ$  and  $89.427^\circ$ . The values of the tangents in this range lie between .01 and 100. If the rule is closed and you are reading on the C or D scales, the tangent is less than 1; when you are reading on the CI or DI scales, the tangent is greater than 1. For very small angles less than  $0.573^\circ$ , refer to Appendix A.

*Verify the following:*

- |                              |                              |                                |
|------------------------------|------------------------------|--------------------------------|
| 1. $\tan 48.5^\circ = 1.130$ | 4. $\tan 46.2^\circ = 1.043$ | 7. $\tan 53^\circ 18' = 1.342$ |
| 2. $\tan 62^\circ = 1.881$   | 5. $\tan 58.7^\circ = 1.645$ | 8. $\tan 80.6^\circ = 6.04$    |
| 3. $\tan 89.2^\circ = 71.6$  | 6. $\tan 84.8^\circ = 11.02$ | 9. $\tan 87^\circ 30' = 22.9$  |

#### 14.7 The extended T scale

Some slide rules have another direct T scale ranging from  $45^\circ$  to  $84.3^\circ$  which we shall refer to as the "extended" T scale (on Pickett rules, this scale is back-to-back with the regular T scale). If the hairline is set over an angle on this extended scale, the tangent may be read directly on the C scale. For angles on this scale, the tangent lies between 1 and 10. If the angle is greater than  $84.3^\circ$ , the tangent is found in the same manner described in the previous section.

#### Exercise 14-3

- |                        |                          |
|------------------------|--------------------------|
| 1. $\tan 33^\circ =$   | 10. $\tan 67.4^\circ =$  |
| 2. $\tan 17^\circ =$   | 11. $\tan 3.7^\circ =$   |
| 3. $\tan 55^\circ =$   | 12. $\tan 1.64^\circ =$  |
| 4. $\tan 78^\circ =$   | 13. $\tan 14.3^\circ =$  |
| 5. $\tan 2^\circ =$    | 14. $\tan 73.6^\circ =$  |
| 6. $\tan 87^\circ =$   | 15. $\tan 49.4^\circ =$  |
| 7. $\tan 13.4^\circ =$ | 16. $\tan 86.3^\circ =$  |
| 8. $\tan 37.6^\circ =$ | 17. $\tan 89.15^\circ =$ |
| 9. $\tan 51^\circ =$   | 18. $\tan 4^\circ 15' =$ |

19.  $\tan 0^\circ 50' =$

20.  $\tan 11.65^\circ =$

21.  $\tan 80.4^\circ =$

22.  $\tan 37^\circ 20' =$

23.  $\tan 21^\circ 15' =$

24.  $\tan 52.1^\circ =$

25.  $\tan 5^\circ 10' =$

26.  $\tan 85.75^\circ =$

27.  $\tan 2.66^\circ =$

28.  $\tan 43^\circ 40' =$

29.  $\tan 84.05^\circ =$

30.  $\tan 7^\circ 50' =$

### 14.8 The cotangent, secant, and cosecant

To find the cotangent we either locate the angle on the complementary scale of T and read the cotangent on C, or locate the angle on the direct T scale and read the reciprocal on CI.

**Example 1:**  $\cot 68^\circ = ?$

1. Close rule and move HL over  $68^\circ$  on complementary scale of T.
2. Under HL read "404" on C or D.

Complement is on T; hence, result is between 0.1 and 1.0. Answer is **0.404**.

**Example 2:**  $\cot 26^\circ = ?$

Here, the angle may be located directly on T, the tangent will be on C, and the desired cotangent is on CI.

1. Close rule and move HL over  $26^\circ$  on T.
2. Under HL read "205" on CI or DI.

The tangent on C is between 0.1 and 1.0; hence, the reciprocal on CI is between 1 and 10. Answer is **2.05**.

**Example 3:**  $\cot 86^\circ = ?$

First obtain the complement:  $90^\circ - 86^\circ = 4^\circ$ .

1. Close rule and move HL over  $4^\circ$  on ST.
2. Under HL read "699" on C or D.

Complement is on ST; hence, result on C is between .01 and 0.1. Answer is **.0699**.

*Verify the following:*

- |                              |                                |                               |
|------------------------------|--------------------------------|-------------------------------|
| 1. $\cot 54^\circ = 0.727$   | 4. $\cot 33^\circ 30' = 1.511$ | 7. $\cot 87.35^\circ = .0463$ |
| 2. $\cot 70.8^\circ = 0.348$ | 5. $\cot 4^\circ 15' = 13.47$  | 8. $\cot 78.4^\circ = 0.205$  |
| 3. $\cot 12.4^\circ = 4.55$  | 6. $\cot 8.3^\circ = 6.86$     | 9. $\cot 2.44^\circ = 23.5$   |

The secant and cosecant can be evaluated as reciprocals of the cosine and sine respectively.

**Example 4:**  $\sec 52^\circ = ?$

1. Close rule and move HL over  $52^\circ$  on the complementary scale of S. The cosine is now on C (or D), and the secant is on CI (or DI).
2. Under HL read "1624" on CI or DI.

The cosine is between 0.1 and 1.0; hence, reciprocal is between 1 and 10. Answer is **1.624**.

**Example 5:**  $\csc 3^\circ = ?$

We find  $\sin 3^\circ$  and read the reciprocal.

1. Close rule and move HL over  $3^\circ$  on ST. The sine is now on C and the cosecant is on CI (or DI).
2. Under HL read "1911" on CI or DI.

Angle is on ST; hence, sine is between .01 and 0.1 and the reciprocal must be between 10 and 100. Answer is **19.11**.

*Verify the following:*

- |                             |                                |                              |
|-----------------------------|--------------------------------|------------------------------|
| 1. $\sec 28^\circ = 1.133$  | 3. $\csc 14.6^\circ = 3.97$    | 5. $\sec 85.7^\circ = 13.33$ |
| 2. $\sec 62.4^\circ = 2.16$ | 4. $\csc 54^\circ 30' = 1.228$ | 6. $\csc 5.9^\circ = 9.73$   |

#### 14.9 A summary of procedures with the trigonometric scales

The following summary may be useful; note especially the general remarks about decimal point placement.

*To find  $\sin x$  or  $\csc x$ :*

1. Move HL over  $x$  on **S** or **ST**.
2. Under HL read  $\sin x$  on **C**,  $\csc x$  on **CI**.

*To find  $\cos x$  or  $\sec x$ :*

1. Move HL over *complement of  $x$*  on **S** or **ST**.
2. Under HL read  $\cos x$  on **C**,  $\sec x$  on **CI**.

*To find  $\tan x$  or  $\cot x$ :*

*Angles less than  $45^\circ$ :*

1. Move HL over  $x$  on **T** or **ST**.
2. Under HL read  $\tan x$  on **C**,  $\cot x$  on **CI**.

*Angles greater than  $45^\circ$ :*

1. Move HL over *complement of  $x$*  on **T** or **ST**.
2. Under HL read  $\tan x$  on **CI**,  $\cot x$  on **C**.

*To place the decimal point:*

1. When reading from the S or T scale to:
    - a. the C scale, answer is between 0.1 and 1.0 (.XXX).
    - b. the CI scale, answer is between 1 and 10 (X.XX).
  2. When reading from the ST scale to:
    - a. The C scale, answer is between .01 and 0.1 (.0XXX).
    - b. the CI scale, answer is between 10 and 100 (XX.X).
- If rule is closed, readings may also be made on D and DI.

#### Exercise 14-4

All six trigonometric functions are represented in this exercise set.

1.  $\cot 62^\circ =$

9.  $\cot 87.4^\circ =$

2.  $\cot 24^\circ =$

10.  $\sec 85.7^\circ =$

3.  $\cot 34^\circ =$

11.  $\sin 71^\circ 20' =$

4.  $\csc 56^\circ =$

12.  $\csc 17.3^\circ =$

5.  $\csc 23^\circ =$

13.  $\tan 22.3^\circ =$

6.  $\sec 47^\circ =$

14.  $\sin 28.3^\circ =$

7.  $\cot 73.5^\circ =$

15.  $\cos 80.55^\circ =$

8.  $\cot 56.2^\circ =$

16.  $\csc 35.6^\circ =$

17.  $\tan 57.4^\circ =$

18.  $\tan 72.6^\circ =$

19.  $\cot 18.4^\circ =$

20.  $\sin 9^\circ 40' =$

21.  $\csc 72.6^\circ =$

22.  $\cos 86.66^\circ =$

23.  $\sec 81^\circ 30' =$

24.  $\tan 1^\circ 45' =$

25.  $\cos 33^\circ 15' =$

26.  $\sin 46^\circ 25' =$

27.  $\cot 40.7^\circ =$

28.  $\cos 15^\circ =$

29.  $\tan 8.45^\circ =$

30.  $\tan 88.22^\circ =$

31.  $\sin 3^\circ 40' =$

32.  $\cos 58.4^\circ =$

33.  $\cot 63.5^\circ =$

34.  $\sin 6^\circ 27' =$

35.  $\cos 89.13^\circ =$

36.  $\cot 2^\circ 30' =$

37.  $\csc 14.7^\circ =$

38.  $\sin 4.05^\circ =$

39.  $\sec 76.3^\circ =$

40.  $\tan 41.2^\circ =$

41.  $\sin 56^\circ 50' =$

42.  $\cot 27.7^\circ =$

43.  $\sin 79^\circ =$

44.  $\cot 85.8^\circ =$

45.  $\csc 2^\circ 20' =$

46.  $\cos 89.15^\circ =$

47.  $\tan 52^\circ 45' =$

48.  $\sec 72.5^\circ =$

## Chapter 15

# FURTHER OPERATIONS WITH THE TRIGONOMETRIC SCALES

### 15.1 Angles greater than $90^\circ$ ; negative angles

These examples assume familiarity with the algebraic signs of the functions in the various quadrants. The reference angle is the acute angle between the terminal side of the angle and the  $x$  axis. Negative angles are generated clockwise from the positive  $x$  axis.

**Example 1:**  $\sin 217^\circ = ?$

Here, the reference angle is  $217^\circ - 180^\circ = 37^\circ$ . (See Figure 15.1.) The given angle is in the third quadrant; hence, the sine is negative. Thus, we may write:

$$\sin 217^\circ = -\sin 37^\circ$$

Verify that the result is  **$-0.602$** .

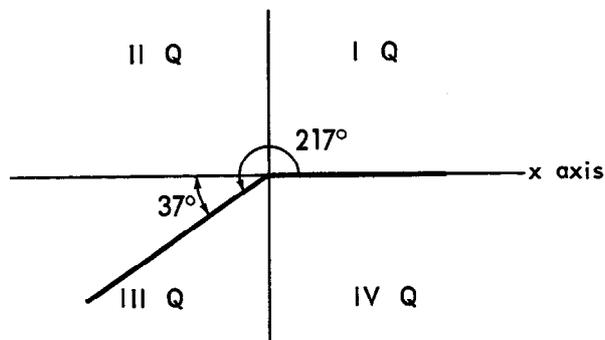


Figure 15.1

**Example 2:**  $\cos 126^\circ = ?$

The reference angle is  $180^\circ - 126^\circ = 54^\circ$ . The given angle is in the second quadrant; hence, the cosine is negative. We may write:

$$\cos 126^\circ = -\cos 54^\circ$$

Verify that the result is **-0.588**.

**Example 3:**  $\tan(-114^\circ) = ?$

The reference angle is  $180^\circ - 114^\circ = 66^\circ$ . (See Figure 15.2.) The given angle is in the third quadrant; hence, the tangent is positive. Therefore, we may write:

$$\tan(-114^\circ) = \tan 66^\circ$$

Verify that the answer is **2.25**.

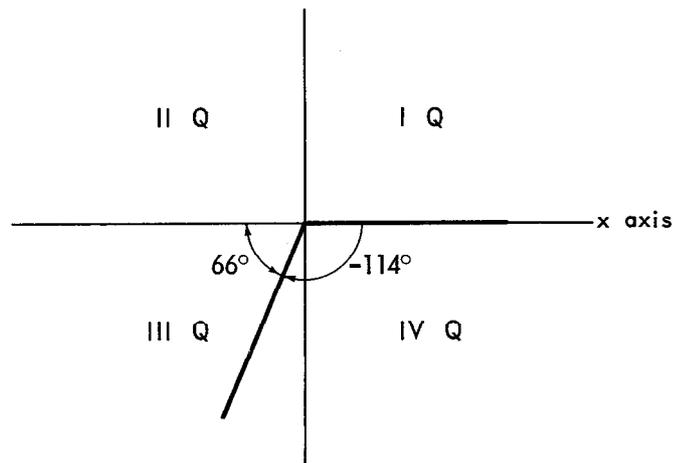


Figure 15.2

### Exercise 15-1

1.  $\sin 148^\circ =$

2.  $\sin 110^\circ =$

3.  $\sin 204^\circ =$

4.  $\cos 123^\circ =$

5.  $\cos 114^\circ =$

6.  $\tan 156^\circ =$

7.  $\tan 280^\circ =$

8.  $\tan 251^\circ =$

9.  $\sin(-136^\circ) =$

10.  $\sin 312^\circ =$

11.  $\cos 137.6^\circ =$

12.  $\cos(-32.4^\circ) =$

13.  $\tan 93.3^\circ =$

16.  $\tan(-112^\circ) =$

14.  $\sin 176^\circ 30' =$

17.  $\sin 477^\circ =$

15.  $\cos 154^\circ 30' =$

18.  $\cos 283.7^\circ =$

## 15.2 The inverse trigonometric functions

Here, we have the reverse problem: given the value of a function, find the corresponding angle. You will recall that “ $\arcsin x$ ” or “ $\sin^{-1} x$ ” are notations which simply mean “angle whose sine is  $x$ .” Similarly, the notations “ $\arctan x$ ” or “ $\tan^{-1} x$ ” mean “angle whose tangent is  $x$ ,” and so forth. We refer to these as “inverse trigonometric functions.”

For example, consider the forms:

a.  $\sin x = 0.245; x = ?$

b.  $\sin^{-1} 0.245 = ?$

c.  $\arcsin 0.245 = ?$

Although different notations are used, all three of these state exactly the same problem: find the angle whose sine is 0.245.

Clearly then, evaluating the inverse functions involves a reversal of the procedures outlined in the previous chapter. In the following examples and exercises, we shall assume that the desired angle is the *smallest positive angle* satisfying the condition.

**Example 1:**  $\sin x = 0.676; x = ?$

1. Close rule and move HL over 676 on C or D. Note that the function is between 0.1 and 1.0; hence, angle is on S.
2. Under HL read **42.5°** on S.

**Example 2:**  $\tan x = .0365; x = ?$

1. Close rule and move HL over 365 on C or D. Function is between .01 and 0.1; hence, angle is on ST.
2. Under HL read **2.09°** on ST.

**Example 3:**  $\cos^{-1} 0.1875 = ?$

1. Close rule and move HL over 1875 on C or D. Function is between 0.1 and 1.0; hence, angle is on S. It is a *cosine* function, therefore, read the *complement*.
2. Under HL read **79.2°** on complementary scale of D.

**Example 4:**  $\arctan 1.8 = ?$

Here, the tangent is between 1 and 10; hence, angle is greater than 45°. It follows, therefore, that the hairline is set on CI and the result is read on the complementary scale of T.

1. Close rule and move HL over 180 on CI or DI.
2. Under HL read **60.95°** on complementary scale of T.

**Example 5:**  $\arcsin(-0.415) = ?$

1. Close rule and move HL over 415 on C or D.
2. Under HL read "24.5°" on S.
3. The function is negative, and we know that the sine is negative in the third and fourth quadrants. Since it has been agreed to choose the smallest positive angle, we select the third quadrant. Hence, the desired angle is  $180^\circ + 24.5^\circ = \mathbf{204.5^\circ}$ .

**Example 6:**  $\tan^{-1}(-0.161) = ?$

1. Close rule and move HL over 161 on C or D.
2. Under HL read "9.15°" on T.
3. The function is negative, and the tangent is negative in the second and fourth quadrants. The smallest positive angle will lie in the second quadrant; hence, result is  $180^\circ - 9.15^\circ = \mathbf{170.85^\circ}$ .

### Exercise 15-2

Find the *smallest positive angle*.

- |                              |                              |
|------------------------------|------------------------------|
| 1. $\sin x = 0.320$ ; $x =$  | 16. $\sin C = 0.950$ ; $C =$ |
| 2. $\sin x = 0.575$ ; $x =$  | 17. $\tan C = 1.303$ ; $C =$ |
| 3. $\sin x = 0.834$ ; $x =$  | 18. $\arctan 4.00 =$         |
| 4. $\tan A = 0.322$ ; $A =$  | 19. $\arcsin .0901 =$        |
| 5. $\tan A = 0.662$ ; $A =$  | 20. $\arctan 0.1875 =$       |
| 6. $\tan x = 0.815$ ; $x =$  | 21. $\arctan 1.13 =$         |
| 7. $\cos A = 0.900$ ; $A =$  | 22. $\arccos 0.764 =$        |
| 8. $\cos A = 0.600$ ; $A =$  | 23. $\arctan 22.9 =$         |
| 9. $\cos B = 0.319$ ; $B =$  | 24. $\arctan 13.2 =$         |
| 10. $\sin x = .0521$ ; $x =$ | 25. $\arccos 0.269 =$        |
| 11. $\sin B = .0378$ ; $B =$ | 26. $\arcsin .0733 =$        |
| 12. $\cos B = .0450$ ; $B =$ | 27. $\arcsin -0.381 =$       |
| 13. $\cos x = 0.891$ ; $x =$ | 28. $\arccos -0.205 =$       |
| 14. $\tan x = .0366$ ; $x =$ | 29. $\arccos -0.474 =$       |
| 15. $\tan x = .0610$ ; $x =$ | 30. $\arctan -0.924 =$       |

- |                          |   |
|--------------------------|---|
| 31. $\arctan .01397 =$   | 42. $\tan^{-1} 10.0 =$                  |
| 32. $\arcsin 0.1095 =$   | 43. $\tan^{-1} 0.667 =$                 |
| 33. $\arctan -0.294 =$   | 44. $\sin^{-1} .0276 =$                 |
| 34. $\cos^{-1} -0.948 =$ | 45. $\cos^{-1} .0279 =$                 |
| 35. $\cos^{-1} .0362 =$  | 46. $\cos^{-1} .01535 =$                |
| 36. $\tan^{-1} 52.5 =$   | 47. $\sin^{-1} -0.245 =$                |
| 37. $\tan^{-1} 2.95 =$   | 48. $\tan^{-1} 1.085 =$                 |
| 38. $\sin^{-1} .0194 =$  | 49. $\cos^{-1} 0.332 =$                 |
| 39. $\sin^{-1} .0734 =$  | 50. $\tan^{-1} 15.45 =$                 |
| 40. $\tan^{-1} 1.85 =$   | 51. $4\sin^2 A + 5\sin A - 6 = 0; A =$  |
| 41. $\cos^{-1} -.0910 =$ | 52. $6\sin^2 x - 11\sin x + 3 = 0; x =$ |

### 15.3 Combined operations with trigonometric functions

When multiplying or dividing trigonometric functions by other numbers, simply bear in mind that the ST, S, and T scales are directly related to the C scale. In fact, you may think of these scales as “auxiliary C scales” which are merely graduated and labeled in different ways. In multiplication and division, therefore, the ST, S, and T scales operate in the same manner as does the C scale.

**Example 1:**  $15.2 \sin 22^\circ = ?$

1. Set left index of C opposite 152 on D.
2. Move HL over  $22^\circ$  on S. This puts HL over  $\sin 22^\circ$  on C; hence, we have performed the desired multiplication.
3. Under HL read “570” on D.

The angle occurs on the S scale; hence,  $\sin 22^\circ$  is between 0.1 and 1.0. The product, therefore, must be between 1.52 and 15.2.

Answer is **5.70**.

**Example 2:**  $435 \sin 36.5^\circ = ?$   $435 \cos 36.5^\circ = ?$

1. Set right index of C opposite 435 on D.
2. Move HL over  $36.5^\circ$  on S. This multiplies by  $\sin 36.5^\circ$ .

3. Under HL read "259" on D.
4. Move HL over  $36.5^\circ$  on *complementary scale* of S. This multiplies by  $\cos 36.5^\circ$ .
5. Under HL read "350" on D.

Answers are **259** and **350**.

Example 3:  $\frac{26.4}{\cos 52.2^\circ} = ?$

1. Move HL over 264 on D.
2. Slide  $52.2^\circ$  on *complementary scale* of S under HL.
3. Opposite right index of C read "431" on D.

Answer is **43.1**.

Example 4:  $1340 \tan 3.44^\circ = ?$

1. Set left index of C opposite 1340 on D.
2. Move HL over  $3.44^\circ$  on ST.
3. Under HL read "805" on D.

The angle is on ST; hence,  $\tan 3.44^\circ$  is between .01 and 0.1, and the product must be between 13.4 and 134.

Answer is **80.5**.

*Verify the following:*

- |  |   |
|--|---|
| 1. $145 \cos 71.5^\circ = 46.0$          | 4. $75 \tan 28.4^\circ = 40.5$            |
| 2. $8.6 \sin 37^\circ = 5.18$            | 5. $36.4 \sin 1.2^\circ = 0.763$          |
| 3. $\frac{2.72}{\cos 46.2^\circ} = 3.93$ | 6. $\frac{680}{\tan 18^\circ 30'} = 2030$ |

Example 5:  $12.5 \tan 63^\circ = ?$

Evaluate this as  $12.5/\cot 63^\circ$ .

1. Move HL over 125 on D.
2. Slide  $63^\circ$  on complementary scale of T under HL.
3. Opposite right index of C, read "245" on D. Answer is **24.5**.

Example 6:  $6.25 \cot 37^\circ = ?$

Evaluate this as  $6.25/\tan 37^\circ$ . Verify that result is **3.30**.

Example 7:  $\frac{2350 \sin 2.3^\circ}{4.66 \tan 17.5^\circ} = ?$

1. Move HL over 235 on D.
2. Slide 466 on C under HL.
3. Move HL over 2.3° on ST.
4. Slide 17.5° on T under HL.
5. Opposite right index of C read "642" on D.

To place the decimal point, first observe that  $\sin 2.3^\circ$  is about .04, and  $\tan 17.5^\circ$  is about 0.3 (this can be done by simply glancing from the ST and T scales to the C scale). We may now write:

$$\frac{2350 \times \sin 2.3^\circ}{4.66 \times \tan 17.5^\circ} \approx \frac{2000 \times .04}{5 \times 0.3} = \frac{16}{0.3} \approx 50. \quad \text{Answer must be } \mathbf{64.2}.$$

### Exercise 15-3

- |  |  |
|--|--|
| 1. $12.8 \sin 46^\circ =$                                  | 14. $478 \sin 18^\circ 20' =$<br>$478 \cos 18^\circ 20' =$ |
| 2. $64 \sin 33.4^\circ =$                                  | 15. $\frac{36.2}{\sin 44^\circ} =$                         |
| 3. $7.6 \tan 26.2^\circ =$                                 | 16. $\frac{267}{\cos 32^\circ} =$                          |
| 4. $125 \cos 54^\circ =$                                   | 17. $1275 \tan 3^\circ 30' =$                              |
| 5. $1450 \sin 2.25^\circ =$                                | 18. $\frac{2.85}{\sin 2.34^\circ} =$                       |
| 6. $6.57 \tan 34.3^\circ =$                                | 19. $\frac{345}{\tan 32.4^\circ} =$                        |
| 7. $12.8 \sin 11^\circ 30' =$                              | 20. $\frac{72.5}{\sin 52^\circ} =$                         |
| 8. $630 \sin 62.3^\circ =$<br>$630 \cos 62.3^\circ =$      | 21. $\frac{2400}{\cos 58.2^\circ} =$                       |
| 9. $12.65 \sin 22.4^\circ =$<br>$12.65 \cos 22.4^\circ =$  | 22. $\frac{4.73}{\cos 15^\circ} =$                         |
| 10. $7500 \sin 12.45^\circ =$<br>$7500 \cos 12.45^\circ =$ | 23. $\frac{66.7}{\sin 18^\circ 15'} =$                     |
| 11. $344 \sin 37.2^\circ =$<br>$344 \cos 37.2^\circ =$     | 24. $6.8 \tan 71^\circ =$                                  |
| 12. $115 \sin 127^\circ =$<br>$115 \cos 127^\circ =$       | 25. $2450 \tan 53.5^\circ =$                               |
| 13. $9.60 \sin 206.2^\circ =$<br>$9.60 \cos 206.2^\circ =$ |  |

26.  $19.6 \cot 34^\circ =$

27.  $480 \cot 18.4^\circ =$

28.  $\frac{36.2 \sin 15^\circ}{2.75 \sin 38^\circ} =$

29.  $\frac{\sin 46^\circ 30'}{.026 \sin 12^\circ 30'} =$

30.  $\frac{175 \cos 48^\circ}{\sin 22^\circ} =$

31.  $10.8 \cos 63^\circ \sin 23^\circ =$

32.  $47.5 \sin 37^\circ \sin 2.4^\circ =$

33.  $\frac{22.6 \sin^2 18.4^\circ}{1.66} =$

34.  $\frac{1}{2}(21.3)(18.4) \sin 13^\circ 15' =$

35.  $\frac{1}{2}(4.66)(7.43) \sin 128.5^\circ =$

36.  $(16.45)(3.08) \cos 33.6^\circ =$

37.  $(12.54)(4.88) \cos 129^\circ =$

38.  $\frac{5600 \tan 3^\circ 30'}{24.4 \tan 65^\circ} =$

39.  $\frac{246 \sin 32.4^\circ \cos 51.5^\circ}{\sin 11.25^\circ} =$

40.  $\frac{6800 \sin 1.65^\circ}{23.5 \tan 31.2^\circ} =$

### 15.4 Radian measurement

Radians and degrees are related as follows:

$$1 \text{ radian} = \frac{180}{\pi} \text{ degrees}$$

Within slide rule accuracy,  $\frac{180}{\pi} = 57.3$ , and we use the approximate relation:

$$1 \text{ radian} = 57.3^\circ$$

The conversion formulas may be stated:

$$\begin{aligned} \text{angle(radians)} &= \text{angle(degrees)} \div 57.3 \\ \text{angle(degrees)} &= \text{angle(radians)} \times 57.3 \end{aligned}$$

Many slide rules have a scribed mark at "573" on C or D (or both).

**Example 1:** Convert  $27.6^\circ$  to radians.

We must divide by 57.3:

1. Move HL over 276 on D.
2. Slide 573 on C under HL.
3. Opposite right index of C, read "482" on D.

Answer:  $27.6^\circ = 0.482$  radians.

**Example 2:**  $\sin 0.64 = ?$

When the degree symbol is omitted, it is understood that the angle is in *radians*. Thus,  $\sin 0.64$  means “the sine of 0.64 radians.” We must, therefore, first convert 0.64 radians to degrees, then find the sine using the S scale.

1. Verify that 0.64 radians = 36.7°.
2. Verify that  $\sin 0.64 = \sin 36.7^\circ = \mathbf{0.598}$ .

*Verify the following:*

- |                                 |                         |
|---------------------------------|-------------------------|
| 1. $28.2^\circ = 0.492$ radians | 5. $\sin 0.44 = 0.426$  |
| 2. $76.5^\circ = 1.336$ radians | 6. $\tan 0.275 = 0.282$ |
| 3. 1.44 radians = $82.5^\circ$  | 7. $\cos 1.1 = 0.454$   |
| 4. 0.38 radians = $21.8^\circ$  | 8. $\sin 2.3 = 0.745$   |

### 15.5 Using the ST scale for radian conversion

For small angles (in the range of ST or smaller), the sine or tangent is approximately equal to the angle itself expressed in radians. Thus,  $\sin 2^\circ$  is about numerically equal to the radian equivalent of  $2^\circ$ . Let us check this on the slide rule:

1. First move HL over  $2^\circ$  on ST, and verify that  $\sin 2^\circ = \mathbf{.0349}$ .
2. Now convert  $2^\circ$  to radians (divide by 57.3), and verify that  $2^\circ = \mathbf{.0349}$  radians.

Therefore, to convert a small angle  $x$  (in the ST range) to radians, we simply find  $\sin x$  using the ST scale, and this also represents the radian equivalent of the angle. For this reason, the ST scale on K & E slide rules is labeled “SRT”; that is, the scale may be used to find *sines*, *radian* equivalents, or *tangents* of small angles.

**Example:** Convert  $2.4^\circ$  to radians.

We proceed exactly as if we are finding  $\sin 2.4^\circ$ :

1. Move HL over  $2.4^\circ$  on ST.
2. Under HL read “419” on C.

Answer:  $2.4^\circ = \mathbf{.0419}$  radians.

### Exercise 15-4

1. Convert to radians:
  - a.  $32^\circ$ , b.  $68.4^\circ$ , c.  $11.4^\circ$ , d.  $145^\circ 30'$ , e.  $223^\circ$

2. Convert to radians (use ST scale):  
a.  $3.65^\circ$ , b.  $4.50^\circ$ , c.  $1.84^\circ$ , d.  $0^\circ 45'$ , e.  $2.76^\circ$
  3. Convert to degrees:  
a.  $2.33$  rad., b.  $0.76$  rad., c.  $1.08$  rad., d.  $5.24$  rad., e.  $0.215$  rad.
  4. Convert to degrees (use ST scale):  
a.  $.062$  rad., b.  $.0345$  rad., c.  $.0175$  rad., d.  $.055$  rad., e.  $.0765$  rad.
5.  $\sin 1.05 =$
  6.  $\tan 0.64 =$
  7.  $\cos 0.22 =$
  8.  $\sin 0.445 =$
  9.  $\sin 2.24 =$
  10.  $\tan 0.36 =$
  11.  $\cos 1.85 =$
  12.  $\sin 1.26 =$
  13.  $\tan 0.175 =$
  14.  $\sin 4 =$
  15.  $\sin(-0.72) =$
  16.  $\cos(-2.9) =$

### 15.6 Formula types

Example 1:  $\sqrt{\frac{36.4}{1 + 2.75 \sin^2 26.5^\circ}} = ?$

1. Verify that  $2.75 \sin^2 26.5^\circ = 0.547$ .
2. Expression may now be evaluated:

$$\sqrt{\frac{36.4}{1 + 0.547}} = \sqrt{\frac{36.4}{1.547}} = 4.85$$

Example 2: Given the formula:  $A = \frac{1}{2} r^2 (\Theta - \sin \Theta)$ .

Find  $A$  when  $r = 12.6$  and  $\Theta = 0.740$  radians.

1. Verify that  $\sin 0.740 = \sin 42.4^\circ = 0.674$ .
2. Formula becomes:

$$A = \frac{1}{2} (12.6)^2 (0.740 - 0.674) = \frac{1}{2} (12.6)^2 (.066) = 5.24.$$

### Exercise 15-5

1.  $53.5(1 - \cos 48^\circ) =$
2.  $\frac{7.22}{3.15 + 2.75 \sin 32^\circ} =$
3.  $\frac{3.65 \cos 36^\circ + 2.88}{\tan 36^\circ} =$

4.  $\frac{82.2}{\cos 27^\circ \cot 15.2^\circ - \sin 27^\circ} =$
5.  $\sqrt{14.7 \cos^2 71^\circ + 4.66} =$
6.  $\sqrt{1 + \pi \sin^2 61^\circ} =$
7.  $\frac{1 - 0.34 \tan 24^\circ}{1 + 0.34/\tan 24^\circ} =$
8.  $\sqrt{\frac{253}{5.64 - 3.22 \sin^2 38.4^\circ}} =$
9.  $\sqrt{(16.3)^2 + (7.44)^2 - 2(16.3)(7.44) \cos 56^\circ} =$
10.  $\sqrt{(23.7)^2 + (38.2)^2 - 2(23.7)(38.2) \cos 118^\circ} =$
11.  $\sin^{-1} \left( \frac{12.8 + 8.44}{12.8 \times 8.44} \right) =$
12.  $\tan^{-1} \left[ \frac{123 - 2.6(4.4)^2}{2.6 \times 4.4} \right] =$
13.  $\cos^{-1} \left[ \frac{(5.7)^2 + (6.2)^2 - (4.8)^2}{2 \times 5.7 \times 6.2} \right] =$

In the following formulas, substitute the given data and evaluate:

14.  $S_n = \frac{1}{2} S(1 - \cos 2\Theta)$   
 a.  $S = 2450, \Theta = 23.6^\circ$ ;    b.  $S = 16,700, \Theta = 35.2^\circ$
15.  $R = \frac{V^2 \sin 2\Theta}{2g}$   
 a.  $V = 1450, \Theta = 18^\circ 20', g = 32.2$   
 b.  $V = 825, \Theta = 28^\circ 30', g = 32.2$
16.  $\tan \Theta = \frac{a \cos \alpha}{a \sin \alpha + g}$  ( $0^\circ < \Theta < 90^\circ$ )  
 a.  $\alpha = 28.2^\circ, a = 12.4, g = 32.2$ . Find  $\Theta$ .  
 b.  $\alpha = 34.9^\circ, a = 21.6, g = 32.2$ . Find  $\Theta$ .
17.  $n = \frac{\sin i}{\sin r}$   
 a.  $i = 36.2^\circ, r = 25.4^\circ$ ;    b.  $i = 56.2^\circ, r = 32.8^\circ$
18.  $A = \frac{1}{2} n r^2 \sin \left[ \frac{360^\circ}{n} \right]$   
 a.  $n = 14, r = 15.1$ ;    b.  $n = 17, r = 7.6$

19.  $A = n r^2 \tan \left[ \frac{180^\circ}{n} \right]$   
 a.  $n = 17, r = 8.2$ ; b.  $n = 13, r = 21.5$
20.  $c = \sqrt{a^2 + b^2 - 2ab \cos C}$   
 a.  $a = 6.3, b = 4.7, C = 42^\circ$   
 b.  $a = 12.7, b = 16.2, C = 123^\circ 30'$
21.  $R = \frac{T \cos \Theta}{1 - \cos \phi \sin \Theta}$   
 a.  $T = 350, \Theta = 31^\circ, \phi = 58^\circ$   
 b.  $T = 2400, \Theta = 27.5^\circ, \phi = 36.3^\circ$
22.  $\phi = \tan^{-1} \frac{2d/r}{(1/r)^2 - 1} \quad (0^\circ < \phi < 90^\circ)$   
 a.  $r = 0.36, d = 3.47$ ; b.  $r = 0.22, d = 4.65$
23.  $T = \frac{\tan \alpha + f/\cos \Theta}{1 - f \tan \alpha/\cos \Theta}$   
 a.  $f = 0.15, \alpha = 23^\circ, \Theta = 32^\circ$   
 b.  $f = 0.24, \alpha = 18^\circ, \Theta = 27^\circ$
24.  $\tan 2\Theta = \frac{S_{xy}}{\frac{1}{2}(S_y - S_x)} \quad (0^\circ < \Theta < 45^\circ)$   
 a.  $S_{xy} = 7500, S_x = 12,400, S_y = 15,240$ . Find  $\Theta$ .  
 b.  $S_{xy} = 13,500, S_x = 18,400, S_y = 22,600$ . Find  $\Theta$ .
25.  $A = \frac{A_0 \sin \Theta}{1 - (w/w_n)^2}$   
 a.  $A_0 = 4.65, \Theta = 54^\circ, w = 340, w_n = 825$   
 b.  $A_0 = 21.2, \Theta = 48^\circ, w = 460, w_n = 710$
26.  $T = \frac{4R \sin \frac{1}{2}\Theta}{\Theta + \sin \Theta}$   
 a.  $R = 8.64, \Theta = 0.42$  radians; b.  $R = 21.7, \Theta = 1.22$  radians
27.  $F_1 = F_2 \left[ \frac{\tan \Theta \cos \alpha}{\cos \beta} + \tan \beta \sin \alpha \right]$   
 a.  $F_2 = 340, \Theta = 18^\circ, \alpha = 22^\circ, \beta = 26^\circ$   
 b.  $F_2 = 76, \Theta = 21.5^\circ, \alpha = 25.6^\circ, \beta = 29.5^\circ$
28.  $A = \frac{1}{2}\pi r^2 - \left[ x \sqrt{r^2 - x^2} + r^2 \sin^{-1} \left( \frac{x}{r} \right) \right] \quad \left( \sin^{-1} \left( \frac{x}{r} \right) \text{ in radians} \right)$   
 a.  $r = 11.7, x = 6.2$ ; b.  $r = 6.25, x = 3.75$
29.  $S = 2\pi b^2 + \frac{2\pi ab}{e} \sin^{-1} e \quad (\sin^{-1} e \text{ in radians})$   
 a.  $a = 8.5, b = 6.4, e = 0.66$   
 b.  $a = 14.6, b = 13.5, e = 0.38$

$$30. \cos \frac{1}{2}A = \sqrt{\frac{\sin s \sin(s-a)}{\sin b \sin c}} \quad (0^\circ < A < 180^\circ)$$

a.  $a = 60^\circ, b = 135^\circ, c = 110^\circ, s = 152.5^\circ$ . Find  $A$ .

b.  $a = 124^\circ, b = 48^\circ, c = 145^\circ, s = 158.5^\circ$ . Find  $A$ .

$$31. Y = e \sec \left[ \frac{L}{2} \sqrt{\frac{P}{EI}} \right]$$

a.  $E = 29 \times 10^6, P = 22,000, I = 14.8, L = 87, e = 0.75$

b.  $E = 29 \times 10^6, P = 18,500, I = 21.5, L = 120, e = 0.85$

## Chapter 16

# THE RIGHT TRIANGLE

### 16.1 Relations between the sides and angles

If  $A$  and  $B$  represent the acute angles of a right triangle, we let  $a$  and  $b$  denote the sides opposite  $A$  and  $B$  respectively, and let  $c$  denote the hypotenuse (Figure 16.1).

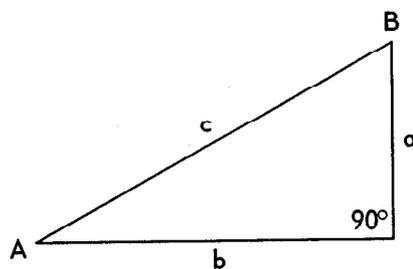


Figure 16.1

In the following section, we shall make use of the relation  $A + B = 90^\circ$ , and we must also recall the definitions:

$$\sin A = \frac{a}{c}$$

$$\sin B = \frac{b}{c}$$

$$\cos A = \frac{b}{c}$$

$$\cos B = \frac{a}{c}$$

$$\tan A = \frac{a}{b}$$

$$\tan B = \frac{b}{a}$$

### 16.2 Solution of the triangle using the function definitions

A right triangle is uniquely determined if an acute angle and a side are known, or if two sides are known. Given the known parts, the problem is to find the desired unknown parts. By choosing the proper function (sine, cosine, or tangent), we can always find the unknown parts in terms of the known. It is usually worthwhile to make a sketch roughly to scale as a check on the reasonableness of the solution.

**Example 1:** Given  $A = 34.2^\circ$  and  $c = 346$ . Find sides  $a$  and  $b$  (Figure 16.2).

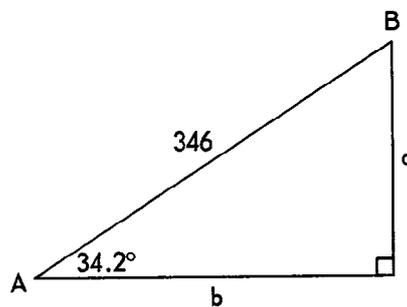


Figure 16.2

1. Write:  $a = 346 \sin 34.2^\circ$ ;  $b = 346 \cos 34.2^\circ$ .
2. Verify that  $a = \mathbf{194.5}$  and  $b = \mathbf{286}$ .

**Example 2:** Given  $c = 46$  and  $a = 21$ . Find remaining parts (Figure 16.3).

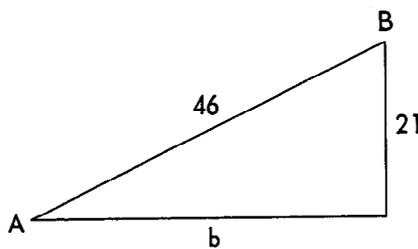


Figure 16.3

1. Write:  $\sin A = \frac{21}{46}$ . Verify that  $A = \mathbf{27.2^\circ}$ .
2. Obtain  $B = 90^\circ - 27.2^\circ = \mathbf{62.8^\circ}$ . This may also be read on the complementary scale in step (1).
3. Write:  $b = 46 \cos 27.2^\circ$ . Verify that  $b = \mathbf{40.9}$ .

**Example 3:** Given  $a = 4.2$  and  $b = 3.4$ . Find remaining parts (Figure 16.4).

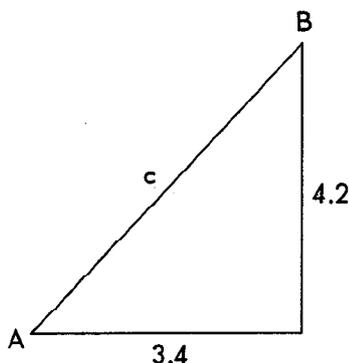


Figure 16.4

When the two legs are given, we first find the *smaller* acute angle using the tangent relation. The smaller angle will be less than  $45^\circ$ ; hence, the answer will appear directly on the T (or ST) scale. In this example,  $B$  is the smaller angle (it is opposite the shorter side).

1. Write:  $\tan B = \frac{3.4}{4.2}$ . Verify that  $B = 39^\circ$ .
2. Write:  $A = 90^\circ - 39^\circ = 51^\circ$ . Again, this may be read on the complementary scale in step (1).
3. Write:  $\sin 51^\circ = \frac{4.2}{c}$ , or  $c = \frac{4.2}{\sin 51^\circ}$ . Verify that  $c = 5.4$ .

### Exercise 16-1

Solve the right triangles (find remaining parts):

- |                              |                              |
|------------------------------|------------------------------|
| 1. $A = 26^\circ, c = 73$    | 5. $a = 36, b = 59$          |
| 2. $A = 56.2^\circ, c = 315$ | 6. $a = 1500, b = 830$       |
| 3. $a = 35, c = 52$          | 7. $B = 48.4^\circ, c = 425$ |
| 4. $b = 145, c = 220$        | 8. $a = 6.32, c = 9.40$      |

### 16.3 Solution using law of sines

Although any right triangle may be solved by the methods of the previous section, the law of sines provides a more efficient slide rule solution. For the right triangle, the law of sines takes the form:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin 90^\circ}{c}$$

This relationship will give a complete solution for all cases except where the known parts are the legs,  $a$  and  $b$ .

The proportion may be solved using the S and D scales. If the hairline is moved over an angle on S, it locates the sine of the angle on C; thus, although angles are set and read on S, the proportion is actually being solved on the C-D scales.

**Example 1:** Given  $A = 38^\circ$  and  $a = 480$ . Find remaining parts (Figure 16.5).

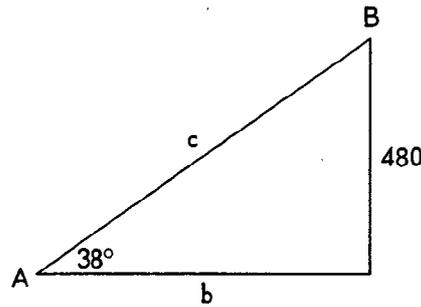


Figure 16.5

First, find  $B$  from the complementary relation:  $B = 90^\circ - 38^\circ = 52^\circ$ . Law of sines becomes:

$$\frac{S}{D}: \frac{\sin 38^\circ}{480} = \frac{\sin 52^\circ}{b} = \frac{\sin 90^\circ}{c}$$

The slide rule settings for this proportion are illustrated in Figure 16.6.

1. Move HL over 480 on D.
2. Slide  $38^\circ$  on S under HL. This sets up the known ratio.
3. Move HL over  $52^\circ$  on S. Under HL read "615" on D.
4. Move HL over  $90^\circ$  on S. Under HL read "780" on D.

Answers are:  $B = 52^\circ, b = 615, c = 780$ .

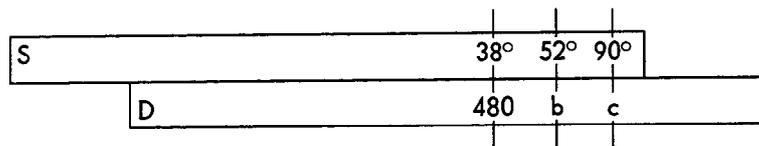


Figure 16.6

**Example 2:** Given  $c = 4.3$  and  $a = 1.8$ . Find remaining parts (Figure 16.7).

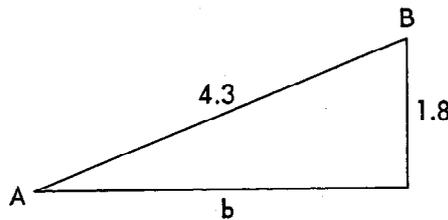


Figure 16.7

Writing the known ratio first, the law of sines becomes:

$$\frac{S}{D} : \frac{\sin 90^\circ}{4.3} = \frac{\sin A}{1.8} = \frac{\sin B}{b}$$

1. Move HL over 43 on D.
2. Slide  $90^\circ$  on S under HL. This sets up the known ratio.
3. Move HL over 18 on D. Under HL read  $24.8^\circ$  on S. This is angle  $A$ .
4. Under HL read  $65.2^\circ$  on complementary scale of S. This is angle  $B$ .
5. Move HL over  $65.2^\circ$  on S. Under HL read "391" on D.

Answers are:  $A = 24.8^\circ$ ,  $B = 65.2^\circ$ ,  $b = 3.91$

**Example 3:** Given  $a = 5$ ,  $c = 62$ . Find remaining parts (Figure 16.8).

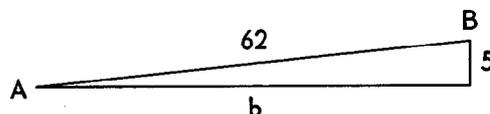


Figure 16.8

Law of sines becomes:

$$\frac{S}{D} : \frac{\sin 90^\circ}{62} = \frac{\sin A}{5} = \frac{\sin B}{b}$$

1. Move HL over 62 on D.
2. Slide  $90^\circ$  on S under HL.
3. Move HL over 5 on D. Under HL read either  $53.7^\circ$  on S or  $4.62^\circ$  on ST. From the sketch it is evident that the smaller angle is the proper choice; hence,  $A = 4.62^\circ$ . From the complementary relation, we have  $B = 90^\circ - 4.62^\circ = 85.38^\circ$ .
4. Move HL over  $85^\circ$  on S (this is all the accuracy we have at this end of the scale). Under HL read "618" on D.

Answers are:  $A = 4.62^\circ$ ,  $B = 85.38^\circ$ ,  $b = 61.8$ .

**Example 4:** Given  $A = 50^\circ$ ,  $b = 7.2$ . Find remaining parts (Figure 16.9).

From the complementary relation:  $B = 90^\circ - 50^\circ = 40^\circ$ . Law of sines becomes:

$$\frac{S}{D} : \frac{\sin 40^\circ}{7.2} = \frac{\sin 50^\circ}{A} = \frac{\sin 90^\circ}{c}$$

1. Move HL over 72 on D.
2. Slide  $40^\circ$  on S under HL.
3. Move HL over  $50^\circ$  on S. Under HL read "858" on D.
4. Now  $90^\circ$  on S (right index of C) is off-scale; hence, we look to the left index of C. Opposite left C index read "1120" on D.

Answers are:  $B = 40^\circ$ ,  $a = 8.58$ ,  $c = 11.20$ .

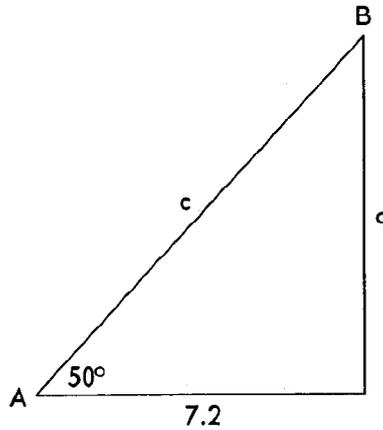


Figure 16.9

**Example 5:** Given  $A = 63^\circ$  and  $a = 12$ . Find remaining parts.

First, find angle  $B$ :  $B = 90^\circ - 63^\circ = 27^\circ$ . Law of sines becomes:

$$\frac{S}{D}: \frac{\sin 63^\circ}{12} = \frac{\sin 27^\circ}{b} = \frac{\sin 90^\circ}{c}$$

1. Move HL over 12 on D.
2. Slide  $63^\circ$  on S under HL.
3. Move HL over  $90^\circ$  on S. Under HL read "1348" on D.
4. Now,  $27^\circ$  on S is off-scale; hence, we interchange indexes (slide left index of C under HL).
5. Move HL over  $27^\circ$  on S. Under HL read "612" on D.

Answers are:  $B = 27^\circ$ ,  $b = 6.12$ ,  $c = 13.48$ .

In example 5, the slide would have been in better position, if the ratio had been set up on S-DF rather than S-D. Note, however, that this requires using both sides of the rule. Also, keep in mind that if the ratio is originally set up on S-D, the proportion must be solved entirely on these scales. Similarly, if it is set up on S-DF, it must be solved entirely on S-DF. In either case, off-scale readings must be handled by interchanging indexes.

From the foregoing examples, you will observe that it is not really necessary to write down the law of sines. Since opposite parts of the triangle are located opposite each other on the slide rule (sides on D, angles on S), you can solve directly from the sketch itself.

### Exercise 16-2

Solve the following right triangles. Use the law of sines.

- |                              |                                |
|------------------------------|--------------------------------|
| 1. $A = 25^\circ$ , $a = 32$ | 3. $B = 35^\circ$ , $a = 6.3$  |
| 2. $A = 53^\circ$ , $c = 44$ | 4. $B = 61^\circ$ , $b = 15.6$ |

- |                                |                                |
|--------------------------------|--------------------------------|
| 5. $A = 47.3^\circ, b = 320$   | 13. $a = 1700, c = 3650$       |
| 6. $a = 23, c = 37$            | 14. $B = 86.2^\circ, a = 3.45$ |
| 7. $a = 51, c = 83$            | 15. $B = 31.4^\circ, b = 6.76$ |
| 8. $b = 840, c = 2800$         | 16. $a = 12.2, c = 160$        |
| 9. $B = 17.5^\circ, c = 4.24$  | 17. $B = 5.1^\circ, b = 1.85$  |
| 10. $a = 3.65, c = 58$         | 18. $b = 5600, c = 8250$       |
| 11. $A = 4.2^\circ, a = 5.55$  | 19. $A = 15.3^\circ, c = 366$  |
| 12. $B = 13.6^\circ, c = 4000$ | 20. $A = 71^\circ, b = 21.6$   |

#### 16.4 A special method for the case with 2 legs given

This case arises frequently in vector applications, and the following examples may be worth your special attention. The method involves finding the smaller acute angle using the tangent relation, then completing the solution with the law of sines.

**Example 1:** Given  $a = 4.4$  and  $b = 6.3$ . Find remaining parts (Figure 16.10).

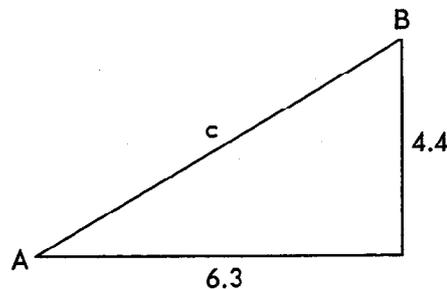


Figure 16.10

Angle  $A$  is the smaller angle, and we write the tangent relation:

$$\tan A = \frac{4.4}{6.3}, \text{ or, in proportion form: } \frac{\tan A}{4.4} = \frac{1}{6.3}$$

Solving the proportion:

1. Set right index of C opposite 63 on D.
2. Move HL over 44 on D.
3. Under HL read  $A = 34.9^\circ$  on T. (Read  $B = 55.1^\circ$  on complementary T scale.)

Applying law of sines:  $\frac{\sin 34.9^\circ}{4.4} = \frac{\sin 90^\circ}{c}$

4. HL is already over 44 on D; hence, simply slide  $34.9^\circ$  on S under HL.
5. Move HL over  $90^\circ$  on S (C index). Under HL read  $c = 7.7$  on D.

The foregoing procedure may be summarized as follows:

*To solve a right triangle with 2 legs given:*

1. Set **C index** opposite *longer* leg on **D**.
2. Move HL over *shorter* leg on **D**.
3. Under HL read *smaller* angle on **T**. (Larger angle may be read on complementary T scale.)
4. Slide the *smaller* angle on **S** under HL.
5. Opposite **C index** read *hypotenuse* on **D**.

**Example 2:** Given  $a = 345$  and  $b = 270$ . Find remaining parts (Figure 16.11).

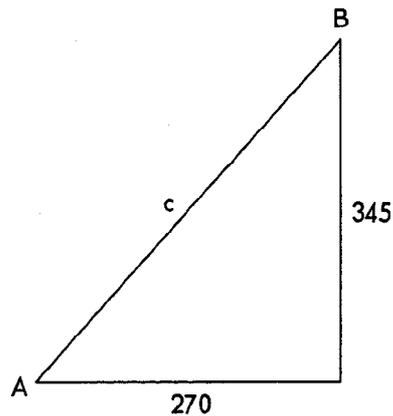


Figure 16.11

In this case, angle  $B$  is the smaller angle.

1. Set right index of C opposite 345 on D. (This is the longer leg.)
2. Move HL over 270 on D.
3. Under HL read  $B = 38.1^\circ$  on T. (Read  $A = 51.9^\circ$  on complementary scale.)
4. Slide  $38.1^\circ$  on S under HL.
5. Opposite C index read  $c = 438$  on D.

**Example 3:** Given  $a = 7.25$  and  $b = 14.60$ . Find remaining parts.

Angle  $A$  is the smaller angle.

1. Set left C index opposite 146 on D.
2. Move HL over 725 on D.
3. Under HL read  $A = 26.4^\circ$  on T. (Read  $B = 63.6^\circ$  on complementary T scale.)

4. Slide  $26.4^\circ$  on S under IIL.
5. Opposite C index read  $c = 16.30$  on D.

*Verify the following:*

1. Given  $a = 65$ ,  $b = 39$ ; verify that  $A = 59^\circ$ ,  $B = 31^\circ$ ,  $c = 75.8$ .
2. Given  $a = 37.5$ ,  $b = 72.4$ ; verify that  $A = 27.4^\circ$ ,  $B = 62.6^\circ$ ,  $c = 81.5$ .
3. Given  $a = 1500$ ,  $b = 840$ ; verify that  $A = 60.8^\circ$ ,  $B = 29.2^\circ$ ,  $c = 1720$ .

**Example 4:** Given  $a = 24$ ,  $b = 260$ . Find remaining parts (Figure 16.12).

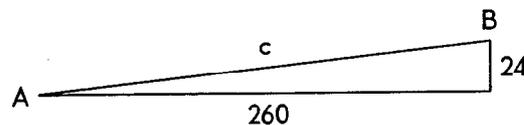


Figure 16.12

1. Set right index of C over 260 on D.
2. Move HL over 24 on D.
3. Under HL read  $42.7^\circ$  on T. However, from the sketch it is evident that angle  $A$  cannot be this large. A rough estimate shows us that  $\tan A$  is about .09; hence, reading should be made on ST. Under HL read  $A = 5.29^\circ$  on ST. (Angle  $B = 90^\circ - 5.29^\circ = 84.71^\circ$ .) Now if we try to slide  $5.29^\circ$  on S under HL, we are referred back to the same position on ST; therefore, when the smaller angle is in the ST range this procedure does not show any slide rule difference between the hypotenuse and the longer leg. This very slight difference does show up if we use the law of sines with the larger angle:

$$\frac{\sin 84.71^\circ}{260} = \frac{\sin 90^\circ}{c}$$

4. Move HL over 260 on D. Slide  $84.71^\circ$  ( $85^\circ$ ) on S under HL.
5. Opposite  $90^\circ$  on S (C index), read  $c = 261$  on D.

### Exercise 16-3

Solve the following right triangles:

- |                        |                           |
|------------------------|---------------------------|
| 1. $a = 12$ , $b = 17$ | 4. $a = 750$ , $b = 1245$ |
| 2. $a = 27$ , $b = 16$ | 5. $a = 9.5$ , $b = 5.8$  |
| 3. $a = 16$ , $b = 41$ | 6. $a = 35$ , $b = 3$     |

7.  $a = 6.5, b = 4.2$

13.  $a = 635, b = 580$

8.  $a = 12.5, b = 18.6$

14.  $a = 1450, b = 3100$

9.  $a = 230, b = 175$

15.  $a = 25.2, b = 420$

10.  $a = 3.6, b = 41$

16.  $a = .075, b = .0266$

11.  $a = 10.75, b = 12.34$

17.  $a = 45.5, b = 112.5$

12.  $a = 0.29, b = 5.06$

18.  $a = 28,500, b = 26,500$

### 16.5 Complex numbers

The complex number  $(a + bj)$ , where  $j = \sqrt{-1}$ , may be represented in the complex plane by the point  $P(a, b)$  as illustrated in Figure 16.13.

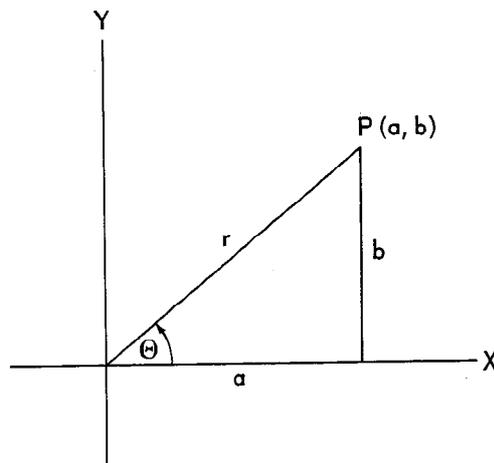


Figure 16.13

At times, it is desirable to represent the point  $P$  in terms of the polar coordinates  $(r, \theta)$ , and from Figure 16.13 it is clear that the following relationships hold:

$$a = r \cos \theta; \quad b = r \sin \theta$$

Therefore, we may write:

$$a + bj = r(\cos \theta + j \sin \theta)$$

We refer to the left side as the *rectangular* form; the expression on the right is called the *polar* form. The polar form is often indicated by the compact notation:  $r \angle \theta$ . Clearly, complex numbers may be used to represent vector quantities, and this is one of their important applications.

It is evident that changing from polar to rectangular form, and vice versa, simply involves solving a right triangle with hypotenuse and side given, or with two legs given.

**Example 1:** Change  $47/38^\circ$  to rectangular form (Figure 16.14).

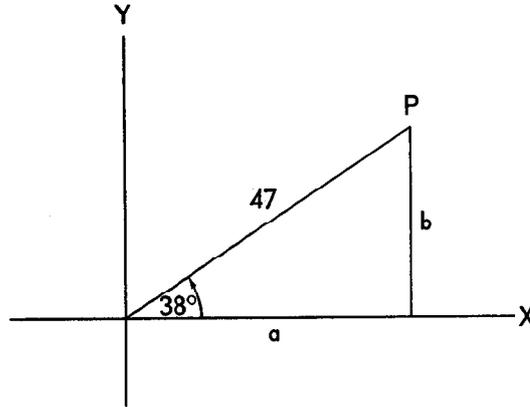


Figure 16.14

Verify that  $a = 47 \cos 38^\circ = 37.0$ ;  $b = 47 \sin 38^\circ = 28.9$   
 Therefore,  $47/38^\circ = 37.0 + 28.9j$ .

**Example 2:** Change  $(37 + 24j)$  to polar form (Figure 16.15).

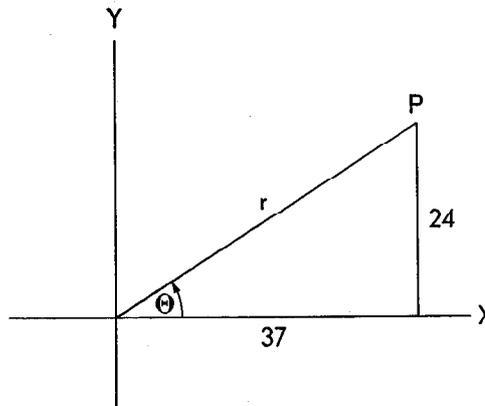


Figure 16.15

1. Set right index of C opposite 37 on D.
2. Move HL over 24 on D. Under HL read  $\Theta = 33^\circ$  on T.
3. Slide  $33^\circ$  on S under HL. Opposite right index of C, read  $r = 44.1$  on D.

Answer:  $37 + 24j = 44.1/33^\circ$ .

**Example 3:** Change  $(-6.3 + 8.2j)$  to polar form (Figure 16.16).

1. Set right index of C opposite 82 on D.
2. Move HL over 63 on D. Under HL read  $\beta = 37.5^\circ$  on T. From the figure, it follows that  $\Theta = 90^\circ + 37.5^\circ = 127.5^\circ$ .

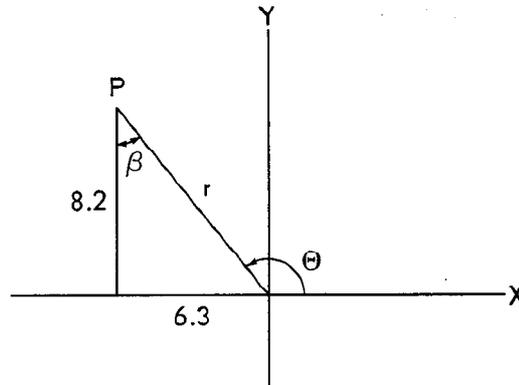


Figure 16.16

3. Slide  $37.5^\circ$  on S under HL. Opposite C index, read  $r = 10.35$  on D.

Answer  $-6.3 + 8.2j = 10.35 / 127.5^\circ$ .

#### Exercise 16-4

1. Change to rectangular form:

a.  $53 / 32^\circ$ , b.  $34 / 56^\circ$ , c.  $180 / 17^\circ$ , d.  $475 / 68^\circ$ , e.  $10.6 / 27.4^\circ$ ,  
 f.  $670 / 3.6^\circ$ , g.  $4.3 / 134^\circ$ , h.  $93.5 / 152.5^\circ$ , i.  $1060 / 216^\circ$ ,  
 j.  $38.2 / 312.5^\circ$ .

2. Change to polar form:

a.  $(3 + 7j)$ , b.  $(21 + 52j)$ , c.  $(16 + 5j)$ , d.  $(5.6 + 2.3j)$ ,  
 e.  $(210 + 750j)$ , f.  $(365 + 23j)$ , g.  $(-27 + 4j)$ , h.  $(-4.55 - 6.26j)$ ,  
 i.  $(-0.82 - 0.22j)$ , j.  $(3400 - 720j)$ .

3. To multiply two complex numbers, we may use the product formula:

$$r_1 / \theta_1 \times r_2 / \theta_2 = r_1 r_2 / \theta_1 + \theta_2.$$

Represent the following products in *rectangular* form:

a.  $3.2 / 23^\circ \times 2.4 / 31^\circ$ , b.  $2.8 / 14^\circ \times 6.5 / 48^\circ$ ,  
 c.  $48.2 / 37.2^\circ \times 0.585 / 21.1^\circ$ , d.  $5.33 / 18.2^\circ \times 7.05 / 4.5^\circ$ ,  
 e.  $23.6 / 52.7^\circ \times 3.68 / 73.3^\circ$ , f.  $21 / 110^\circ \times 36 / 140^\circ$ .

4. The division formula for complex numbers may be stated:

$$r_1 / \theta_1 \div r_2 / \theta_2 = r_1 / r_2 / \theta_1 - \theta_2.$$

Represent the following quotients in *rectangular* form:

a.  $16 / 57^\circ \div 2.7 / 17^\circ$ , b.  $230 / 96^\circ \div 64 / 32^\circ$ ,  
 c.  $125 / 133^\circ \div 344 / 68^\circ$ , d.  $5.35 / 214^\circ \div 7.40 / 59^\circ$ ,  
 e.  $37.5 / 317^\circ \div 4.66 / 85^\circ$ , f.  $45.3 / 27.4^\circ \div 6.24 / 68.2^\circ$ .

## Chapter 17

# THE OBLIQUE TRIANGLE

### 17.1 Relations between the sides and angles

If  $A$ ,  $B$ , and  $C$  represent the angles of a triangle, we let  $a$ ,  $b$ , and  $c$  denote the sides opposite  $A$ ,  $B$ , and  $C$ , respectively (Figure 17.1).

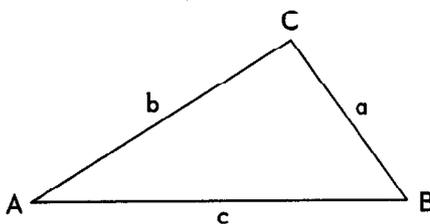


Figure 17.1

The following important relations hold for the general triangle:

*Sum of the internal angles:*  $A + B + C = 180^\circ$

*Law of sines:*  $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

*Law of cosines:*  $a^2 = b^2 + c^2 - 2bc \cos A$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

We shall also make use of the identities:

$$\begin{aligned}\sin x &= \sin(180^\circ - x) \\ \cos x &= -\cos(180^\circ - x)\end{aligned}$$

### 17.2 Three parts must be known

As with the right triangle, we are concerned here with finding the unknown parts of the triangle in terms of the known parts. For the oblique triangle, we must know three parts, at least one of which is a side. Thus, we may be given two angles and a side, two sides and an angle, or three sides.

In the following sections, slide rule procedures are described for handling these various cases.

### 17.3 Complete solution using the law of sines

If we are given two angles and a side, or two sides and an angle opposite one of the sides, the triangle may be solved completely using the law of sines.

**Example 1:** Given  $a = 45$ ,  $A = 52^\circ$ , and  $B = 62^\circ$ .

Find the remaining parts (Figure 17.2).

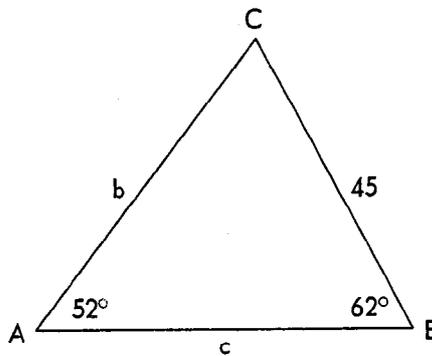


Figure 17.2

First, find angle  $C$ :  $C = 180^\circ - (52^\circ + 62^\circ) = 66^\circ$ .

Law of sines now becomes:

$$\frac{S}{D}: \frac{\sin 52^\circ}{45} = \frac{\sin 62^\circ}{b} = \frac{\sin 66^\circ}{c}$$

1. Move HL over 45 on D.
2. Slide  $52^\circ$  on S under HL. This sets up the known ratio.
3. Move HL over  $62^\circ$  on S. Under HL read  $b = 50.4$  on D.
4. Move HL over  $66^\circ$  on S. Under HL read  $c = 52.1$  on D.

**Example 2:** Given  $C = 70^\circ$ ,  $a = 36$ , and  $c = 45$ .

Find remaining parts (Figure 17.3).

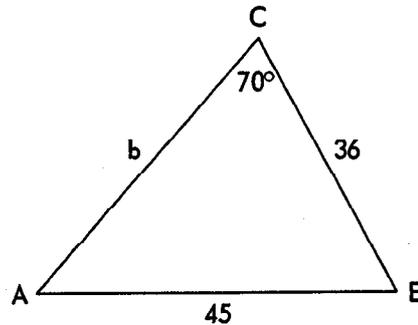


Figure 17.3

Applying the law of sines:

$$\frac{S}{D}: \frac{\sin A}{36} = \frac{\sin B}{b} = \frac{\sin 70^\circ}{45}$$

1. Move HL over 45 on D.
2. Slide  $70^\circ$  on S under HL. This sets up the ratio.
3. Move HL over 36 on D. Under HL read  $A = 48.7^\circ$  on S.  
Solve for angle  $B$ :  $B = 180^\circ - (48.7^\circ + 70^\circ) = 61.3^\circ$ .
4. Move HL over  $61.3^\circ$  on S. Under HL read  $b = 42.0$ .

**Example 3:** Given  $A = 27.2^\circ$ ,  $C = 34.3^\circ$ , and  $b = 137$ .

Find remaining parts (Figure 17.4).

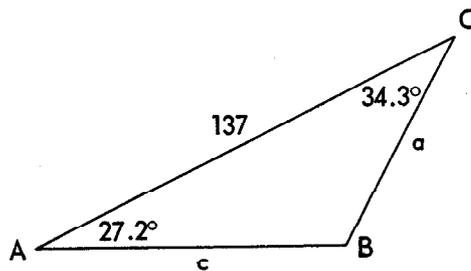


Figure 17.4

First, find angle  $B$ :  $B = 180^\circ - (27.2^\circ + 34.3^\circ) = 118.5^\circ$ .

Law of sines becomes:

$$\frac{\sin 118.5^\circ}{137} = \frac{\sin 27.2^\circ}{a} = \frac{\sin 34.3^\circ}{c}$$

Inasmuch as  $\sin 118.5^\circ = \sin(180^\circ - 118.5^\circ) = \sin 61.5^\circ$ , the sine law relation may be rewritten:

$$\frac{\sin 61.5^\circ}{137} = \frac{\sin 27.2^\circ}{a} = \frac{\sin 34.3^\circ}{c}$$

Now observe that the slide will be in better position if the proportion is set up on the S-DF scales.

1. Move HL over 137 on DF.
2. Slide 61.5° on S under HL. This sets up the ratio.
3. Move HL over 27.2° on S. Under HL read  $a = 71.3$  on DF.
4. Move HL over 34.3° on S. Under HL read  $c = 87.9$  on DF.

### Exercise 17-1

Solve the following triangles.

- |  |  |
|--|--|
| 1. $A = 66^\circ, B = 48^\circ, a = 42$      | 11. $A = 80^\circ 30', B = 56^\circ 15', b = 7.05$ |
| 2. $B = 37^\circ, C = 63^\circ, b = 26$      | 12. $A = 14^\circ 20', C = 38^\circ 30', a = 148$  |
| 3. $a = 6.2, b = 4.6, A = 67^\circ$          | 13. $C = 154^\circ, a = 21.2, c = 27.4$            |
| 4. $a = 85, c = 57, A = 41^\circ$            | 14. $A = 3^\circ 30', C = 58^\circ 10', c = 115$   |
| 5. $B = 125^\circ, b = 16, c = 3.5$          | 15. $A = 42^\circ, a = 73, b = 28$                 |
| 6. $A = 126^\circ, C = 15^\circ, b = 17$     | 16. $b = 2.65, c = 13.7, C = 22.4^\circ$           |
| 7. $A = 75^\circ, B = 62^\circ, c = 10.4$    | 17. $A = 73^\circ, B = 41.6^\circ, a = 21.7$       |
| 8. $B = 63.2^\circ, C = 4.8^\circ, a = 6.44$ | 18. $a = 12.7, c = 1.65, A = 100^\circ$            |
| 9. $a = 17.2, c = 31.3, C = 72^\circ$        | 19. $B = 159^\circ, b = 355, c = 402$              |
| 10. $B = 46^\circ, b = 7.65, c = 5.20$       | 20. $B = 43^\circ 15', C = 119^\circ 30', a = 236$ |

### 17.4 The case with two solutions

**Example:** Given  $A = 45^\circ, a = 16$ , and  $b = 20$ . Find remaining parts.

Since the side opposite the given angle is shorter than the other given side, two triangles are possible (Figure 17.5).

Applying law of sines to triangle  $ABC$ :

$$\frac{S}{D}: \frac{\sin 45^\circ}{16} = \frac{\sin B}{20} = \frac{\sin C}{c}$$

Verify that:  $B = 62^\circ, C = 73^\circ$ , and  $c = 21.6$ .

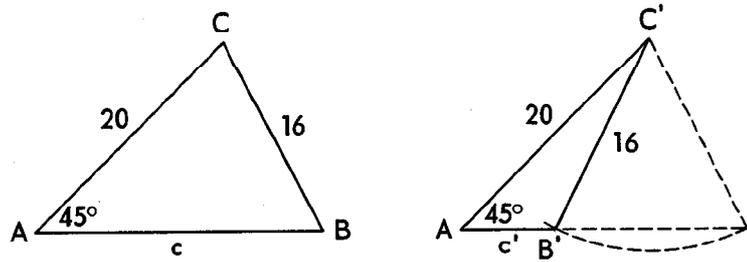


Figure 17.5

Now in triangle  $AB'C'$ , we first find angles  $B'$  and  $C'$ :

$$\begin{aligned} B' &= 180^\circ - B = 180^\circ - 62^\circ = 118^\circ \\ C' &= 180^\circ - (45^\circ + 118^\circ) = 17^\circ \end{aligned}$$

Law of sines becomes:

$$\frac{\sin 45^\circ}{16} = \frac{\sin 118^\circ}{20} = \frac{\sin 17^\circ}{c'}$$

The ratio is already set up on S-D; however,  $17^\circ$  is off-scale on S; hence, we must first interchange indexes. Verify that  $c' = 6.62$ . Summarizing the two solutions:

$$1. B = 62^\circ, C = 73^\circ, c = 21.6; \quad 2. B' = 118^\circ, C' = 17^\circ, c' = 6.62.$$

Note: If the original proportion had been set up on S-DF, it would not have been necessary to interchange indexes.

### Exercise 17-2

Solve the triangles (two solutions possible).

- |  |  |
|--|--|
| 1. $A = 26^\circ, a = 4.5, b = 5.2$    | 5. $A = 43^\circ, a = 48.2, c = 56$      |
| 2. $A = 37^\circ, a = 13, b = 16$      | 6. $C = 5.3^\circ, a = 12.3, c = 2.64$   |
| 3. $A = 41^\circ, a = 22, b = 27$      | 7. $b = 8.65, c = 13.24, B = 14.6^\circ$ |
| 4. $A = 8.4^\circ, a = 2.66, c = 6.23$ | 8. $b = 0.65, c = 1.65, B = 22.6^\circ$  |

### 17.5 Application of the law of cosines

The law of cosines is useful when we are given two sides and the included angle, or three sides.

**Example 1:** Given  $A = 37^\circ, b = 10, c = 15$ . Find remaining parts (Figure 17.6).